Math 1B, Quiz 6: Solutions

Monday, March 9

1. TRUE OR FALSE (2 pts each). You DO NOT have to show your reasoning if the answer is true. If the answer is false, provide a counterexample (if appropriate).

   (a) If $\sum a_n$ is absolutely convergent then $\sum a_n \cos(n)$ is absolutely convergent.
   True, since $|a_n \cos(n)| \leq |a_n|$, and since $\sum |a_n|$ converges the comparison test implies that $\sum |a_n \cos n|$ also converges. Since the series is absolutely convergent, it is convergent.

   (b) If $a_n > b_n > 0$ and $\sum a_n$ is divergent then $\sum b_n$ is divergent.
   False: $a_n = 1, b_n = 1/n^2$.

   (c) The Ratio Test can be used to show that $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ is convergent.
   False: Applying the Ratio Test to this series will yield 1, which is inconclusive.

2. (2 pts each) Identify each of the following series as absolutely convergent, conditionally convergent, or divergent. You must justify your answers.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$
   $|a_n| = 1/\sqrt{n}$ so the series is not absolutely convergent.
   The series is alternating, $\frac{1}{\sqrt{n}}$ is a decreasing function, and $\lim_{n\to\infty} 1/\sqrt{n} = 0$, so by the Alternating Series test the series is conditionally convergent.

   (b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+3}{2n+4}\right)^n$
   $\lim_{n\to\infty} a_n^{1/n} = \lim_{n\to\infty} \frac{n+3}{2n+4} = 1/2$, so by the Root test the series is absolutely convergent.

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{50^n}{n!}$
   $\lim_{n\to\infty} |a_{n+1}|/|a_n| = \lim_{n\to\infty} \frac{50^{n+1}}{(n+1)!}\frac{n!}{50^n} = \lim_{n\to\infty} \frac{50}{n+1} = 0$, so by the Ratio test the series is absolutely convergent.

Extra Credit

Write $e^{0.06}$ in decimal form as accurately as you can ($\sum_{i=1}^{n} \frac{1}{5i}$ pts for n decimal places).

$e^{0.06} \approx 1.06183654654535962222\ldots$