Math 1B, Quiz 5

Monday, March 2

1. (1 pt each) Decide whether each of the following **series** is convergent or divergent. You DO NOT have to show your work.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{3+5^n}{n^2+7^n}$ converges

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n+1)+\sqrt{n^3+4}}{n^2+5\sin(n^2)}$ diverges

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+\pi}$ converges

2. TRUE OR FALSE (2 pts each). You DO NOT have to show your reasoning if the answer is true. If the answer is false, provide a counterexample.

   (a) If a_n, b_n are sequences with positive terms and $\lim_{n\to\infty} a_n/b_n = 0$ then $\sum_{n=1}^{\infty} a_n$ converges.
   False: $b_n = 1, a_n = 1/n$. If we also have the condition that $\sum_{n=1}^{\infty} b_n$ converges, then the statement is true.

   (b) If $a_n > 0$ and $\sum_{n=1}^{\infty} a_n$ is convergent then $\sum_{n=1}^{\infty} a_n^2$ is convergent.
   True by the Limit Comparison Test, since $\lim_{n\to\infty} a_n^2/a_n = \lim_{n\to\infty} a_n = 0$.

3. (3 pts) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^{1.3}}$ converges or diverges. Show all of your work.

   Compare to $\sum_{n=1}^{\infty} 1/n^{1.1}$:

$$\lim_{n\to\infty} \frac{\ln n}{n^{1.3}}(n^{1.1}) = \lim_{n\to\infty} \frac{\ln n}{n^{0.2}} = 0$$

   The last equality can be proved with L'Hospital's rule, but you did not need to say so to get credit. Then since $\sum_{n=1}^{\infty} 1/n^{1.1}$ converges (by the Integral Test), $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ converges by the Limit Comparison Test.

Extra Credit

Write the fraction $\frac{100}{97}$ in decimal form to as many decimal places as you can (hint: put it in the form $\frac{1}{1-x}$).
($\frac{1}{4n}$ pts for the n-th digit)
Re-write as $\frac{1}{1-\frac{3}{100}} = 1+3/100+(3/100)^2+\ldots = 1+.03+.0009+.000027+\ldots = 1.030927835051546391752577319587628865979$