Math 1B, Quiz 4

Monday, February 23

1. (1 pt each) Decide whether each of the following **sequences** is convergent or divergent. If the sequence is convergent, find the limit.

   (a) $\lim_{n\to\infty} \frac{25^n}{n!} = 0$

   (b) $\lim_{n\to\infty} \frac{1.1^n}{n^{1.1}} = \infty$

   (c) $\lim_{n\to\infty} \frac{\sin n}{n} = 0$

   (d) $\lim_{n\to\infty} \sin n$ DIVERGES

   (e) $\lim_{n\to\infty} \frac{(\ln n)^3}{n^{1/3}} = 0$

   (f) $\lim_{n\to\infty} \frac{n^2 + \ln n}{3n^2 + 2n + \sqrt{n}} = 1/3$

2. (3 pts) Find the limit of the sequence $\lim_{n\to\infty} \sqrt{n^2 + 3n + 1} - n$.

$$\begin{aligned}
\lim_{n\to\infty} \sqrt{n^2+3n+1} - n &= \lim_{n\to\infty} (\sqrt{n^2+3n+1} - n)\frac{\sqrt{n^2+3n+1}+n}{\sqrt{n^2+3n+1}+n} \\
&= \lim_{n\to\infty} \frac{3n+1}{n+\sqrt{n^2+3n+1}} \\
&= \lim_{n\to\infty} \frac{3+1/n}{1+\sqrt{1+3/n+1/n^2}} \\
&= 3/2
\end{aligned}$$

3. (3 pts) Find the sum of the series $\sum_{n=1}^{\infty} \frac{5}{7^n}$.

$$\sum_{n=1}^{\infty} \frac{5}{7^n} = \frac{5}{7}\sum_{n=1}^{\infty}(1/7)^{n-1} = \frac{5}{7}\left(\frac{1}{1-1/7}\right) = 5/6$$

Extra Credit

Mark all statements as true or false (0.1 pt each). Answers will be judged based on their consistency with your other answers rather than according to a theoretical "correct" solution.

1. The sum of the numbers of the true statements is equal to the sum of the numbers of the false statements. **False**
2. All prime-numbered statements are true. **False**
3. The product of the numbers of the false statements is 10. **True**
4. The sum of the numbers of the true statements is prime. **True**
5. All even-numbered statements are false. **False**