Math 1B, Quiz 4
Monday, February 23

1. (1 pt each) Decide whether each of the following sequences is convergent or divergent. If the sequence is convergent, find the limit.

(a) \( \lim_{n \to \infty} \frac{25^n}{n!} = 0 \)
(b) \( \lim_{n \to \infty} \frac{1.1^n}{n^{1.1}} = \infty \)
(c) \( \lim_{n \to \infty} \frac{\sin n}{n} = 0 \)
(d) \( \lim_{n \to \infty} \sin n \) DIVERGES
(e) \( \lim_{n \to \infty} \frac{(\ln n)^3}{n^{1/3}} = 0 \)
(f) \( \lim_{n \to \infty} \frac{n^2 + \ln n}{3n^2 + 2n + \sqrt{n}} = 1/3 \)

2. (3 pts) Find the limit of the sequence \( \lim_{n \to \infty} \sqrt{n^2 + 3n + 1} - n \).

\[
\lim_{n \to \infty} \sqrt{n^2 + 3n + 1} - n = \lim_{n \to \infty} \frac{\sqrt{n^2 + 3n + 1} + n}{\sqrt{n^2 + 3n + 1} + n} \cdot \frac{\sqrt{n^2 + 3n + 1} - n}{\sqrt{n^2 + 3n + 1} - n}
\]
\[
= \lim_{n \to \infty} \frac{3n + 1}{n + \sqrt{n^2 + 3n + 1}}
\]
\[
= \lim_{n \to \infty} \frac{3 + 1/n}{1 + \sqrt{1 + 3/n + 1/n^2}}
\]
\[
= 3/2
\]

3. (3 pts) Find the sum of the series \( \sum_{n=1}^{\infty} \frac{5}{7^n} \).

\[
\sum_{n=1}^{\infty} \frac{5}{7^n} = \frac{5}{7} \sum_{n=1}^{\infty} (1/7)^{n-1} = \frac{5}{7} \left( \frac{1}{1-1/7} \right) = 5/6
\]
Extra Credit

Mark all statements as true or false (0.1 pt each). Answers will be judged based on their consistency with your other answers rather than according to a theoretical “correct” solution.

1. The sum of the numbers of the true statements is equal to the sum of the numbers of the false statements. \textbf{False}

2. All prime-numbered statements are true. \textbf{False}

3. The product of the numbers of the false statements is 10. \textbf{True}

4. The sum of the numbers of the true statements is prime. \textbf{True}

5. All even-numbered statements are false. \textbf{False}