Math 1B, Quiz 2

Monday, February 2

Name:

1. (3 pts) Evaluate the integral $\int \frac{x^2+3}{(x-1)^2}\, dx$.

   Since the degree of the polynomial in the numerator is the same as the degree of the one in the denominator, we first have to carry out long division:

$$\int \frac{x^2+3}{(x-1)^2}\, dx = \int 1 + \frac{2x+2}{(x-1)^2}\, dx$$

   We could keep doing polynomial division from here, or we could use partial fractions to break down the function further:

$$\begin{aligned}
\frac{2x+2}{(x-1)^2} &= \frac{A}{x-1} + \frac{B}{(x-1)^2} \\
\frac{2x+2}{(x-1)^2} &= \frac{A(x-1)}{(x-1)^2} + \frac{B}{(x-1)^2} \\
2x+2 &= A(x-1) + B
\end{aligned}$$

   Solving from here gives $A = 2, B = 4$. Therefore

$$\begin{aligned}
\int \frac{x^2+3}{(x-1)^2}\, dx &= \int 1 + \frac{2}{x-1} + \frac{4}{(x-1)^2}\, dx \\
&= x + 2\ln(x-1) - \frac{4}{x-1}
\end{aligned}$$

2. (3 pts) Evaluate the integral $\int \frac{x+1}{x(x^2+1)}\, dx$.

$$\begin{aligned}
\frac{x+1}{x(x^2+1)} &= \frac{A}{x} + \frac{Bx+C}{x^2+1} \\
x+1 &= A(x^2+1) + (Bx+C)x
\end{aligned}$$

   Solving gives $A = 1, B = -1, C = 1$, so

$$\begin{aligned}
\int \frac{x+1}{x(x^2+1)}\, dx &= \int \frac{1}{x} - \frac{x}{x^2+1} + \frac{1}{1+x^2}\, dx \\
&= \ln(x) - \frac{1}{2}\ln(x^2+1) + \arctan(x) + C
\end{aligned}$$

3. (4 pts) Evaluate the integral $\int \frac{1}{(1-x^2)^{3/2}}\,dx$.

   Substitute $x = \sin\theta, dx = \cos\theta\,d\theta$ to get

$$\begin{aligned}
\int \frac{1}{(1-x^2)^{3/2}}\,dx &= \frac{1}{(1-\sin^2(\theta))^{3/2}}\cos\theta\,d\theta \\
&= \int \frac{1}{\cos^3\theta}\cos\theta\,d\theta \\
&= \int \frac{1}{\cos^2\theta}\,d\theta \\
&= \int \sec^2\theta\,d\theta \\
&= \tan(\theta) \\
&= \tan(\arcsin(x))\,dx \\
&= \frac{x}{\sqrt{1-x^2}}\,dx
\end{aligned}$$

Extra Credit

Mark all statements as true or false (0.1 pt each). Answers will be judged based on their consistency with your other answers rather than according to a theoretical "correct" solution.

1. Exactly one of these statements is false. **False**
2. Exactly two of these statements are false. **True**
3. All three of these statements are false. **False**