# Math 1B: The Ultimate Quiz

## Monday, April 27

1. (4 pts) Find the unique solution to the differential equation $xy' = y + x^2 \sin x$ given the condition $y(\pi) = 0$.

$$\begin{aligned} xy' &= y + x^2 \sin x \\ y' - y/x &= x \sin x \\ I &= e^{\int -1/x} \\ &= e^{-\ln x} \\ &= 1/x \\ y'/x - y/x^2 &= \sin x \\ (y/x)' &= \sin x \\ y/x &= -\cos x + C \\ y &= Cx - x\cos x \\ 0 &= C\pi + \pi \\ C &= -1 \\ y &= -x - x\cos x \end{aligned}$$

2. (6 pts) Find the unique solution to the differential equation $y'' - 2y' + y = 2e^x/x^3$ given the conditions $y(1) = 4e$ and $y(2) = \frac{9}{2}e^2$.

   (a) First get two solutions to the homogeneous equation: $y_1 = e^x, y_2 = xe^x$.

   (b) $W = y_1 y_2' - y_2 y_1' = e^{2x}$.

   (c)

$$\begin{aligned} y &= -y_1 \int \frac{Gy_2}{aW} + y_2 \int \frac{Gy_1}{aW} \\ &= -e^x \int \frac{2e^x x e^x}{x^3 e^{2x}} + xe^x \int \frac{2e^x e^x}{x^3 e^{2x}} \\ &= -e^x \int \frac{2}{x^2} + xe^x \int \frac{2}{x^3} \\ &= -e^x \left(-2/x + C_1\right) + xe^x \left(-1/x^2 + C_2\right) \\ &= C_1 e^x + C_2 x e^x + 2e^x/x - e^x/x \\ &= C_1 e^x + C_2 x e^x + e^x/x \end{aligned}$$

The boundary conditions then give

$$\begin{aligned} C_1 e + C_2 e + e &= 4e \\ C_1 + C_2 &= 3 \\ C_1 e^2 + 2C_2 e^2 + e^2/2 &= \frac{9}{2}e^2 \\ C_1 + 2C_2 &= 4 \\ C_2 &= 1 \\ C_1 &= 2 \end{aligned}$$

The solution is therefore $y = 2e^x + xe^x + e^x/x$.