# Math 1B, Quiz 10

## Monday, April 20

### Terminal Velocity, Revisited

Say we throw a math textbook from the top of the Campanile. Let $g$ be the (downward) accleration due to gravity and let $v_f$ be the terminal velocity of the book. One potential model for the forces acting on the book is $a = \frac{dv}{dt} = g - \frac{g}{v_f}v$, where $v$ is the velocity of the book. Since $v = \frac{dy}{dt}$, we can express the previous equation in terms of y:

$$y'' + \frac{g}{v_f}y' = g$$

1. (3 pts) Use the method of undetermined coefficients to find a solution $y_p$ for $y$.

$$\begin{aligned} y &= Ax^2 + Bx + C \\ y' &= 2Ax + B \\ y'' &= 2A \\ y'' + \frac{g}{v_f}y' &= g \\ 2A + \frac{g}{v_f}2Ax + \frac{g}{v_f}B &= g \\ A &= 0 \\ B &= v_f \\ y &= v_f t \end{aligned}$$

2. (1 pt) Describe in words what your particular solution $y_p$ says about the path of the book. Was it thrown downward, thrown upward, or dropped?

   The book was thrown downward at its terminal velocity, and so it falls at a constant rate over time.

3. (3 pts) Find the general solution to the complementary equation $y'' + \frac{g}{v_f}y' = 0$ by solving the auxiliary equation.

   The auxiliary equation is $r^2 + \frac{g}{v_f}r = 0$, which gives the solutions $r_1 = 0, r_2 = -\frac{g}{v_f}$. The general solution is therefore

$$y = c_1 e^{0t} + c_2 e^{-(g/v_f)t} = c_1 + c_2 e^{-(g/v_f)t}$$

4. (1 pt) Find the general solution to the original nonhomogeneous equation.

$$y = c_1 + v_f t + c_2 e^{-(g/v_f)t}$$

5. (2 pts) If the book was dropped (with zero initial velocity) from 93m up, find the unique solution that models the position of the book over time. Your answer should be in terms of $v_f$ and $g$.

$$\begin{aligned} c_1 + c_2 &= 93 \\ v_f - (g/v_f)c_2 &= 0 \\ c_2 &= v_f^2/g \\ c_1 &= 93 - v_f^2/g \end{aligned}$$

## Extra Credit

(1 gold star) Each answer will be graded based on its consistency with your other answers rather than according to a theoretical "correct" solution.

1. What is the probability that you will answer both of these questions correctly by guessing randomly?
   (a) 1/16
   (b) **1/8**
   (c) 1/8
   (d) 1/4
2. What is the probability that you will answer both of these questions *incorrectly* by guessing randomly?
   (a) 1/4
   (b) **3/8**
   (c) 9/16
   (d) 9/16