Math 1B, Quiz 1

Solutions

1. (3 pts) Evaluate the integral $\int x^{1/3}\ln(x)\,dx$.

   Let $u = \ln(x), du = \frac{1}{x}\,dx$, $dv = x^{1/3}\,dx, v = \frac{3}{4}x^{4/3}$. Then

$$\begin{aligned}\int x^{1/3}\ln(x)\,dx &= \frac{3}{4}x^{4/3}\ln(x) - \frac{3}{4}\int x^{4/3}\frac{1}{x}\,dx \\ &= \frac{3}{4}x^{4/3}\ln(x) - \frac{3}{4}\int x^{1/3}\,dx \\ &= \frac{3}{4}x^{4/3}\ln(x) - \frac{9}{16}x^{4/3} + C\end{aligned}$$

2. (3 pts) Evaluate the integral $\int \sin^3(x)\cos^3(x)\,dx$.

   Solution 1: Rewrite $\sin^2(x)$ as $(1-\cos^2(x))$ and substitute $u = \cos(x), du = -\sin(x)\,dx$. Then

$$\begin{aligned}\int \sin^3(x)\cos^3(x)\,dx &= \int \sin(x)(1-\cos^2(x))\cos^3(x)\,dx \\ &= -\int (1-u^2)u^3\,du \\ &= \int u^5 - u^3\,du \\ &= u^6/6 - u^4/4 + C \\ &= \cos^6(x)/6 - \cos^4(x)/4 + C\end{aligned}$$

   Solution 2: Rewrite $\cos^2(x)$ as $(1-\sin^2(x))$ and substitute $u = \sin(x), du = \cos(x)\,dx$. Then

$$\begin{aligned}\int \sin^3(x)\cos^3(x)\,dx &= \int \sin^3(x)(1-\sin^2(x))\cos(x)\,dx \\ &= \int u^3(1-u^2)\,du \\ &= \int u^3 - u^5\,du \\ &= u^4/4 - u^6/6 + C \\ &= \sin^4(x)/4 - \sin^6(x)/6 + C\end{aligned}$$

   These two answers differ only by the constant term at the end (as you can check by substituting $\cos^2(x) = 1 - \sin^2(x)$ in the first answer).

3. (3 pts) Evaluate the integral $\int \sin^2(x)\,dx$.

   Use the fact that $\cos(2x) = 1 - 2\sin^2(x)$ to get that $\sin^2(x) = \frac{1-\cos(2x)}{2}$, and make this substitution.

$$\begin{aligned}
\int \sin^2(x)\,dx &= \int \frac{1-\cos(2x)}{2}\,dx \\
&= \frac{1}{2}(\int dx - \int \cos(2x)\,dx) \\
&= \frac{1}{2}(x - \frac{1}{2}\sin(2x) + C) \\
&= \frac{1}{2}x - \frac{1}{4}\sin(2x) + C
\end{aligned}$$

Extra Credit

Mark all statements as true or false (0.1 pt).

1. This statement is true.

Either “true” or “false” would work as an answer.