Practice Final
Wednesday, May 6

1. Evaluate the integral $\int \cos x \ln \sin x \, dx$.

2. Evaluate the integral $\int \dfrac{2}{(x^2+1)(x+1)}$.

3. Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \dfrac{\sqrt{n+1}}{2n+1}(3x-1)^n$.

4. Solve the initial-value problem $e^x y' = 1 + y, y(0) = -1$.

5. Find the general solution to the equation $y'' + y = \dfrac{1}{\sin x}$

6. True or False: you do not have to show your work if the answer is true, but give a counterexample if the answer is false.

   (a) If $\{a_n\}$ is any sequence and $\lim_{n\to\infty} b_n = 0$, then $\lim_{n\to\infty} a_n b_n = 0$.

   (b) If $\lim_{n\to\infty} |a_{n+1}|/|a_n| = 1$ then $\sum_{n=1}^{\infty} a_n$ converges conditionally.

   (c) If $a_n, b_n > 0$ and $\lim_{n\to\infty} a_n/b_n = 0$ and $\sum_{n=1}^{\infty} b_n$ diverges then $\sum_{n=1}^{\infty} a_n$ diverges.

   (d) If $a_n > a_{n+1} > 0$ for all n then $\lim_{n\to\infty} a_n$ exists.

7. Mark each integral or series as convergent or divergent. You do not have to show your work.

   (a) $\displaystyle\int_0^{\infty} \frac{1}{\sqrt{|x-3|}} \, dx$

   (b) $\displaystyle\int_1^{\infty} \frac{1}{x^2} \, dx$

   (c) $\displaystyle\int_1^{\infty} \frac{\sqrt{x} + e^{-x}}{x + \ln x} \, dx$

   (d) $\displaystyle\int_0^1 \frac{1}{\sin x} \, dx$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3 + 3^n}{n!}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$