17.2: Nonhomogeneous Linear Equations

Friday, April 17

Recap: Second-order Linear

Find solutions to the following differential equations:

1. $y'' + 2y' = 0; y(0) = 0, y'(0) = 3$
2. $y'' + 2y' + 3y = 0; y(0) = -1, y'(0) = 2$
3. $y'' + 2y' + y = 0; y(0) = 3, y'(0) = 0$
4. $y'' - 4y' + 9y = 0; y(0) = 1, y(3) = 4$
5. $y'' - 4y' + 4y = 0; y(0) = y(2) = 0$
6. $y'' - 4y' - 5y = 0; y(0) = 1, y(1) = 2$

Nonhomogeneous Linear Equations

Let y_p be a solution to $ay'' + by' + cy = G(x)$, where a, b, c are constants.

1. If y_c solves $ay'' + by' + cy = 0$, show that $y_p + y_c$ is another solution to the original problem.
2. If y_q is another solution to $ay'' + by' + cy = G(x)$, show that $y_p - y_q$ is a solution to $ay'' + by' + cy = 0$.
3. If y_1 solves $ay'' + by' + cy = G(x)$ and y_2 solves $ay'' + by' + cy = H(x)$, show that $y_1 + y_2$ solves $ay'' + by' + cy = G(x) + H(x)$.

So to solve an equation of the form $ay'' + by' + cy = G(x) + H(x)$,

1. Find any y_g that solves $ay'' + by' + cy = G(x)$.
2. Find any y_h that solves $ay'' + by' + cy = H(x)$.
3. Find all y_c that solves $ay'' + by' + cy = 0$.
4. All solutions will be of the form $y_g + y_h + y_c$, for some y_c.

More advice! This is from the table on page 1153 of Stewart:

1. If $G(x) = e^{kx}P(x)$, try $y_p = e^{kx}Q(x)$, where $deg(P) = deg(Q)$.
2. If $G(x) = e^{kx}P(x)\cos(mx)$ or $e^{kx}P(x)\sin(mx)$, try $y_p(x) = e^{kx}Q(x)\cos(mx) + e^{kx}R(x)\sin(mx)$.
3. If you do the above and get a solution to the complementary equation, try mulitplying y_p by x or x^2.

Exercises

Find solutions to the following differential equations (the homogenous equations are the same as the ones at the start of the worksheet):

1. $y'' + 2y' = x^2 + 1$
2. $y'' + 2y' + 3y = 3e^{2x}$
3. $y'' + 2y' + y = 2\sin x$
4. $y'' - 4y' + 9y = xe^x$
5. $y'' - 4y' + 4y = x^2 + e^x$
6. $y'' - 4y' - 5y = x + \cos 2x$
7. $y'' - 3y = e^{2x}\sin x$; $y(0) = 2, y'(0) = 0$
8. $y'' - y' = x^2 + x - 1$; $y(0) = 1, y(2) = 1$

Some More Complex Numbers

Using the formula $e^{i\theta} = \cos\theta + i\sin\theta$, write the following numbers in the form $a + bi$ and plot them on the complex plane.

1. $e^{i\pi/2}$
2. $e^{i\pi/4}$
3. $2e^{-i\pi/3}$
4. $\frac{1}{3}e^{i\pi}$
5. $e^{2+i\pi/6}$
6. $e^{-1-i\pi}$

1. If $z = re^{i\theta}$, then write z^2, z^3, and $1/z$ in polar form.
2. Find all solutions to $z^6 = 1$ by putting z in the form $re^{i\theta}$. Plot them in the complex plane.
3. Define the absolute value of $z = a + bi$ as $|z| = \sqrt{a^2 + b^2}$. If $z = re^{i\theta}$, then find $|z|$, $|z^2|$, and $|1/z|$.