17.2: Nonhomogeneous Linear Equations
Friday, April 17

Recap: Second-order Linear

Find solutions to the following differential equations:

1. \( y'' + 2y' = 0; y(0) = 0, y'(0) = 3 \)
2. \( y'' + 2y' + 3y = 0; y(0) = -1, y'(0) = 2 \)
3. \( y'' + 2y' + y = 0; y(0) = 3, y'(0) = 0 \)
4. \( y'' - 4y' + 9y = 0; y(0) = 1, y(3) = 4 \)
5. \( y'' - 4y' + 4y = 0; y(0) = y(2) = 0 \)
6. \( y'' - 4y' - 5y = 0; y(0) = 1, y(1) = 2 \)

Nonhomogeneous Linear Equations

Let \( y_p \) be a solution to \( ay'' + by' + cy = G(x) \), where \( a, b, c \) are constants.

1. If \( y_c \) solves \( ay'' + by' + cy = 0 \), show that \( y_p + y_c \) is another solution to the original problem.
2. If \( y_q \) is another solution to \( ay'' + by' + cy = G(x) \), show that \( y_p - y_q \) is a solution to \( ay'' + by' + cy = 0 \).
3. If \( y_1 \) solves \( ay'' + by' + cy = G(x) \) and \( y_2 \) solves \( ay'' + by' + cy = H(x) \), show that \( y_1 + y_2 \) solves \( ay'' + by' + cy = G(x) + H(x) \).

So to solve an equation of the form \( ay'' + by' + cy = G(x) + H(x) \),

1. Find any \( y_g \) that solves \( ay'' + by' + cy = G(x) \).
2. Find any \( y_h \) that solves \( ay'' + by' + cy = H(x) \).
3. Find all \( y_c \) that solves \( ay'' + by' + cy = 0 \).
4. All solutions will be of the form \( y_g + y_h + y_c \), for some \( y_c \).

More advice! This is from the table on page 1153 of Stewart:

1. If \( G(x) = e^{kx}P(x) \), try \( y_p = e^{kx}Q(x) \), where \( \text{deg}(P) = \text{deg}(Q) \).
2. If \( G(x) = e^{kx}P(x)\cos(mx) \) or \( e^{kx}P(x)\sin(mx) \), try \( y_p(x) = e^{kx}Q(x)\cos(mx) + e^{kx}R(x)\sin(mx) \).
3. If you do the above and get a solution to the complementary equation, try multiplying \( y_p \) by \( x \) or \( x^2 \).
Exercises

Find solutions to the following differential equations (the homogenous equations are the same as the ones at the start of the worksheet):

1. \( y'' + 2y' = x^2 + 1 \)
2. \( y'' + 2y' + 3y = 3e^{2x} \)
3. \( y'' + 2y' + y = 2 \sin x \)
4. \( y'' - 4y' + 9y = xe^x \)
5. \( y'' - 4y' + 4y = x^2 + e^x \)
6. \( y'' - 4y' - 5y = x + \cos 2x \)
7. \( y'' - 3y = e^{2x} \sin x; \ y(0) = 2, y'(0) = 0 \)
8. \( y'' - y' = x^2 + x - 1; \ y(0) = 1, y(2) = 1 \)

Some More Complex Numbers

Using the formula \( e^{i\theta} = \cos \theta + i \sin \theta \), write the following numbers in the form \( a + bi \) and plot them on the complex plane.

1. \( e^{i\pi/2} \)
2. \( e^{i\pi/4} \)
3. \( 2e^{-i\pi/3} \)
4. \( \frac{1}{3}e^{i\pi} \)
5. \( e^{2+i\pi/6} \)
6. \( e^{-1-i\pi} \)

1. If \( z = re^{i\theta} \), then write \( z^2, z^3 \), and \( 1/z \) in polar form.
2. Find all solutions to \( z^6 = 1 \) by putting \( z \) in the form \( re^{i\theta} \). Plot them in the complex plane.
3. Define the absolute value of \( z = a + bi \) as \( |z| = \sqrt{a^2 + b^2} \). If \( z = re^{i\theta} \), then find \( |z|, |z|^2 \), and \( |1/z| \).