11.10: More Taylor Series

Friday, March 20

Recap

Find the intervals of convergence of the following power series:

1. $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n^2 \cdot 2^n}$
2. $\sum_{n=1}^{\infty} \frac{(1-x)^n}{3^n}$
3. $\sum_{n=1}^{\infty} \frac{2^n(x-1)^n}{5^n\sqrt{n}}$
4. $\sum_{n=1}^{\infty} \frac{(4-3x)^n}{n}$
5. $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{3^n}$
6. $\sum_{n=1}^{\infty} \frac{(2-5x)^n}{n \cdot 3^n}$

Polynomial Fitting

1. Find a parabola $P(x) = ax^2 + bx + c$ that goes through the points $(-1, 1), (0, 0)$, and $(1, 2)$.
2. Find a parabola $P(x)$ such that $P(0) = 0$, $P'(0) = -3$, and $P''(0) = 5$.
3. Find a cubic polynomial $Q(x)$ such that $Q(0) = 0, Q'(0) = -3, Q''(0) = 5$, and $Q'''(0) = -1$. How does its graph compare with the graph of the parabola in the previous question?
4. Find a parabola $P(x)$ such that $P(2) = 0$, $P'(2) = 1$, and $P''(2) = -1$ (Hint: write it as $a(x-2)^2 + b(x-2) + c$ rather than $ax^2 + bx + c$. How does this help?)
5. What is the derivative of $f(x) = x^3$ at $x = 0$? The second derivative? Third? Fourth?

Taylor Series: Using Derivatives

Compute the Taylor series for the following functions up to the x^3 term. Graph the functions and the polynomial approximations.

1. $\ln x$ around $x = 1$
2. $\ln x$ around $x = 2$
3. $1/\sqrt{x}$ around $x = 1$
4. $1/\sqrt{x}$ around $x = 4$
5. $\cos x$ around $x = \pi/2$
6. $\tan x$ around $x = 0$

Taylor Series: Using Other Taylor Series

1. Compute the Taylor series for $e^x \sin(x)$ around $x = 0$ and around $x = 1$ up to the x^4 term.
2. Compute the Taylor series for $\dfrac{\cos x}{1 - x}$ around $x = 0$ up to the x^4 term.