Recap

Find the intervals of convergence of the following power series:

1. \[ \sum_{n=1}^{\infty} \frac{(x-3)^n}{n^2 \cdot 2^n} \]
2. \[ \sum_{n=1}^{\infty} \frac{(1-x)^n}{3^n} \]
3. \[ \sum_{n=1}^{\infty} \frac{2^n(x-1)^n}{5^n \sqrt{n}} \]
4. \[ \sum_{n=1}^{\infty} \frac{(4-3x)^n}{n} \]
5. \[ \sum_{n=1}^{\infty} \frac{(2x-1)^n}{3^n} \]
6. \[ \sum_{n=1}^{\infty} \frac{(2-5x)^n}{n \cdot 3^n} \]

Polynomial Fitting

1. Find a parabola \( P(x) = ax^2 + bx + c \) that goes through the points \((-1,1),(0,0),(1,2)\).
2. Find a parabola \( P(x) \) such that \( P(0) = 0, \ P'(0) = -3, \) and \( P''(0) = 5. \)
3. Find a cubic polynomial \( Q(x) \) such that \( Q(0) = 0, \ Q'(0) = -3, \ Q''(0) = 5, \) and \( Q'''(0) = -1. \) How does its graph compare with the graph of the parabola in the previous question?

4. Find a parabola \( P(x) \) such that \( P(2) = 0, \ P'(2) = 1, \) and \( P''(2) = -1 \) (Hint: write it as \( a(x-2)^2 + b(x-2) + c \) rather than \( ax^2 + bx + c \). How does this help?)

5. What is the derivative of \( f(x) = x^3 \) at \( x = 0? \) The second derivative? Third? Fourth?
Taylor Series: Using Derivatives

Compute the Taylor series for the following functions up to the $x^3$ term. Graph the functions and the polynomial approximations.

1. $\ln x$ around $x = 1$
2. $\ln x$ around $x = 2$
3. $1/\sqrt{x}$ around $x = 1$
4. $1/\sqrt{x}$ around $x = 4$
5. $\cos x$ around $x = \pi/2$
6. $\tan x$ around $x = 0$

Taylor Series: Using Other Taylor Series

1. Compute the Taylor series for $e^x \sin(x)$ around $x = 0$ and around $x = 1$ up to the $x^4$ term.
2. Compute the Taylor series for $\frac{\cos x}{1 - x}$ around $x = 0$ up to the $x^4$ term.