11.10: More Taylor Series

Friday, March 20

Recap

Find the intervals of convergence of the following power series:

1. $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n^2 \cdot 2^n}$: $[1,5]$
2. $\sum_{n=1}^{\infty} \frac{(1-x)^n}{3^n}$: $(-2,4)$
3. $\sum_{n=1}^{\infty} \frac{2^n(x-1)^n}{5^n\sqrt{n}}$: $[-3/2, 7/2)$
4. $\sum_{n=1}^{\infty} \frac{(4-3x)^n}{n}$: $(1, 5/3]$
5. $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{3^n}$: $(-1,2)$
6. $\sum_{n=1}^{\infty} \frac{(2-5x)^n}{n \cdot 3^n}$: $(-1/5, 1]$

Polynomial Fitting

1. Find a parabola $P(x) = ax^2 + bx + c$ that goes through the points $(-1,1), (0,0)$, and $(1,2)$.

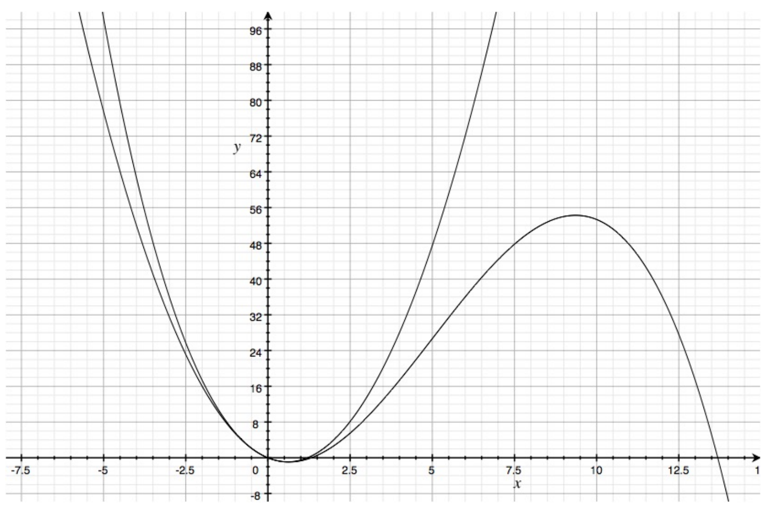
   $P(x) = \frac{3}{2}x^2 + \frac{1}{2}x$.

2. Find a parabola $P(x)$ such that $P(0) = 0$, $P'(0) = -3$, and $P''(0) = 5$.

   $P(x) = \frac{5}{2}x^2 - 3x$.

3. Find a cubic polynomial $Q(x)$ such that $Q(0) = 0, Q'(0) = -3, Q''(0) = 5$, and $Q'''(0) = -1$. How does its graph compare with the graph of the parabola in the previous question?

   $Q(x) = \frac{-1}{6}x^3 + \frac{5}{2}x^2 - 3x$. As the graph below shows, the two functions are most similar at $x = 0$.



4. Find a parabola $P(x)$ such that $P(2) = 0$, $P'(2) = 1$, and $P''(2) = -1$ (Hint: write it as $a(x-2)^2 + b(x-2) + c$ rather than $ax^2 + bx + c$. How does this help?)

   $P(x) = \frac{-1}{2}(x-2)^2 + (x-2)$.

5. What is the derivative of $f(x) = x^3$ at $x = 0$? The second derivative? Third? Fourth?

   First derivative: 0

   Second derivative: 0

   Third derivative: 6

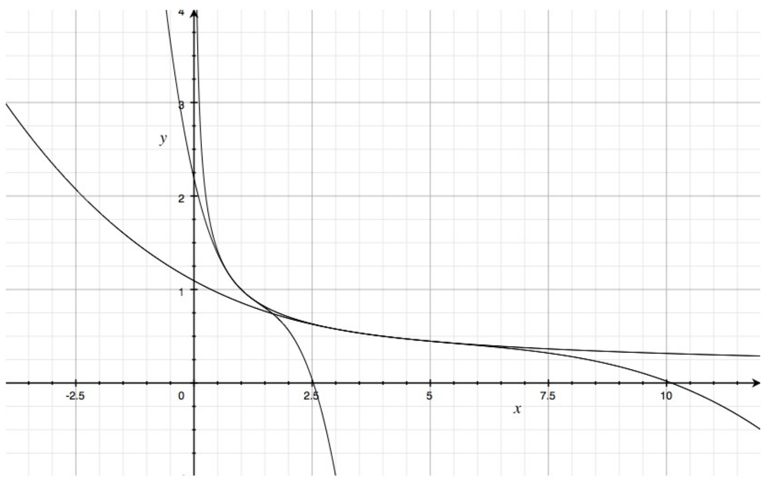
   Fourth derivative: 0

Taylor Series: Using Derivatives

Compute the Taylor series for the following functions up to the x^3 term. Graph the functions and the polynomial approximations.

1. $\ln x$ around $x = 1$: $(x-1) - (x-1)^{/}2 + (x-1)^3/3$
2. $\ln x$ around $x = 2$: $\ln(2) + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2 + \frac{1}{24}(x-2)^3$
3. $1/\sqrt{x}$ around $x = 1$: $1 - \frac{1}{2}(x-1) + \frac{3}{8}(x-1)^2 - \frac{5}{16}(x-1)^3$
4. $1/\sqrt{x}$ around $x = 4$: $\frac{1}{2} - \frac{1}{16}(x-4) + \frac{3}{256}(x-4)^2 - \frac{5}{2048}(x-4)^3$
5. $\cos x$ around $x = \pi/2$: $-(x - \pi/2) + \frac{1}{6}(x - \pi/2)^3$
6. $\tan x$ around $x = 0$: $x + \frac{x^3}{3}$

Below is a graph for $y = 1/\sqrt{x}$ with the cubic approximations at $x = 1$ and $x = 4$:



Taylor Series: Using Other Taylor Series

1. Compute the Taylor series for $e^x \sin(x)$ around $x = 0$ and around $x = 1$ up to the x^4 term.

   Around $x = 0$: $e^x \sin x = x + x^2 + x^3/3 + \ldots$

2. Compute the Taylor series for $\dfrac{\cos x}{1-x}$ around $x = 0$ up to the x^4 term.

   Around $x = 0$: $\frac{\cos x}{1-x} = 1 + x + x^2/2 + x^3/2 + 13x^4/24 + \ldots$