17.2: Nonhomogeneous Equations, continued

Wednesday, April 22

Terminal Velocity

An object is dropped (with zero initial velocity) from the top of the Campanile (94 meters). The object's height y as a function of the time t since it was dropped can be expressed by the following differential equation:

$$y''(t) + \frac{g}{v_t}y'(t) = g$$

Here, g is the acceleration due to gravity near the Earth's surface. v_t is the terminal velocity of the object– the velocity at which the forces of gravity and air resistance cancel each other out. We will set "up" as the positive direction and "down" as the negative, so both g and v_t are negative constants.

1. If we set $g = -10m/s^2$ and $v_t = -30m/s^2$, the equation becomes $y'' + \frac{1}{3}y' = -10$. Given the intial conditions from the first paragraph, solve for $y(t)$.

2. Do the same, but keeping the constants g and v_t in place of the values -10 and -30.

3. What is the velocity of the object (y') as a function of t?

4. Suppose the object were thrown downward with an initial velocity of $-40m/s$. What would the new solution for $y(t)$ look like? Describe what happens to the book in qualitative terms.

The Method of Undetermined Coefficients

Determine the form of the trial solution y_p to the following differential equations:

1. $y'' + y' + 3y = x^3 + x - 1$
2. $y'' + 3y' = \sin 2x$
3. $y'' + 3y' + 2y = e^{5x}$
4. $y'' + 2y' + y = (x+1)\sin 3x$
5. $y'' + y' + 3y = e^{2x}\cos 3x$
6. $y'' + 2y' - 3y = xe^x \sin 2x$

Sometimes you might inadvertently get a solution to the complementary equation instead. Try multiplying by x or x^2...

1. $y'' + 2y' = x^2 + 3$
2. $y'' + 3y' = 4$
3. $y'' - 3y' + 2y = e^x$
4. $y'' + 4y = \sin 2x$
5. $y'' = 1$
6. $y'' + 2y' + y = e^{-x}$

Find the general solutions for each of the previous six problems. For problem #N, find the unique solution with the initial conditions $y(0) = 1$, $y'(0) = N$.