Integration by Parts
Wednesday, January 21

Tips

- "L-I-A-T-E"

  When in doubt, a good heuristic is to choose u to be the first type of function in the following list:

  1. L-Logartithmic functions $(\ln(x), \log_2(x))$.
  2. I-Inverse trig functions $(\arcsin(x), \arccos(x), \arctan(x))$.
  3. A-Algebraic functions $(x^2 + 2x, 1/x, \sqrt{1+x^2})$.
  4. T-Trig functions $(\sin(x), \cos(x), \tan(x))$.
  5. E-Exponential functions $(e^{2x}, 5^x)$.

- If that doesn't lead to a clear choice for u, instead choose dv so that it can be easily integrated.

  Example: If we want to evaluate $\int x^3/\sqrt{1+x^2}\,dx$, there is not a clear choice for u since the entire function is algebraic. If we look at choices for dv instead, we can see that although $1/\sqrt{1+x^2}$ might not have a simple antiderivative, $x/\sqrt{1+x^2}$ has $\sqrt{1+x^2}$ as an antiderivative. So we should choose $dv = x/\sqrt{1+x^2}$ and $u = x^2$.

Warm-up

Use integration by parts to evaluate the following integrals:

1. $\int xe^x\,dx$

2. $\int_1^e \ln(x)\,dx$

3. $\int x^3\sqrt{1+x^2}\,dx$

Speed Round

For the following problems, you do not have to evaluate the integral. Simply circle the part of the function that you would set equal to u (dv will implicitly be everything you did not circle). Answer each batch of problems as quickly and as accurately as you can.

Round 1

1. $\int xe^{3x}\,dx$
2. $\int x\sin(5x)\,dx$
3. $\int x^2\ln(x)\,dx$

4. $\int x^2e^{-5x}\,dx$
5. $\int x^2\cos(3x)\,dx$
6. $\int(x^3 + 4)e^{2x}\,dx$

7. $\int \arcsin(x)\,dx$
8. $\int x\arctan(x)\,dx$
9. $\int x\ln(x)\,dx$

Round 2
This time the LIATE rule might not always get you the answer. Write down your choice of u if you cannot circle it properly.

1. $\int x^2 2^x \, dx$

2. $\int \ln(x)/\sqrt{x} \, dx$

3. $\int x^3/\sqrt{1+x^2} \, dx$

4. $\int e^x \sin(2x) \, dx$

5. $\int x^5 e^{x^2} \, dx$

6. $\int \ln(x)/x^2 \, dx$

7. $\int e^{2x} \sin(e^x) \, dx$

8. $\int x \ln^2(x) \, dx$

9. $\int \arctan(x) \, dx$

Round 3
Using integration by parts might not always be the correct (or best) solution. For the following problems, indicate whether you would use integration by parts (with your choices of u and dv), substitution (with your choice of u), or neither.

1. $\int x \ln(x) \, dx$

2. $\int \ln(x)/x \, dx$

3. $\int e^x \, dx$

4. $\int \frac{1}{x \ln(x)} \, dx$

5. $\int \arccos(2x) \, dx$

6. $\int e^{\cos(x)} \sin(x) \, dx$

7. $\int e^x \sin(x) \, dx$

8. $\int x^3 \cos(x^2) \, dx$

9. $\int 1/(1+x^2) \, dx$

10. $\int x \tan^2(x) \, dx$

11. $\int x^5 e^{-x^2} \, dx$

12. $\int x + \sin(x) \, dx$

Bonus
Evaluate $\int_0^1 x^5 e^{-x}$ using integration by parts. Then use the fact that $0 \le \int_0^1 x^5 e^{-x} \le 1$ (why?) to put an upper and lower bound on e.