

# Improper Integrals

Friday, February 6

## Bounding Functions

Put guaranteed upper and lower bounds on the following, with justification:

1.  $\sin(x) + x \cos(x), x \in [2, 5]$
2.  $x^2 + x, x \in [-1, 1]$
3.  $\sqrt{\ln(x) + x}, x \in [4, 12]$
4.  $e^{1/x} + x - x^2, x \in [2, 4]$
5.  $\sin(\sin(\sin(x^5))), x \in [-10, 10]$
6.  $2x \sin(x) - 4x^2 \cos(x) + e^{x^2}, x \in [0, 1]$
7. The total distance a human can run in a day.
8. The combined weight of all people living in California.

## Infinite Intervals

Determine whether each of the following integrals are convergent or divergent:

- |                                       |   |  |
|---------------------------------------|---|--|
| 1. $\int_1^{\infty} \frac{1}{x} dx$   | 5. $\int_1^{\infty} x e^{-x} dx$            | 9. $\int_1^{\infty} \frac{100000}{x^{1.003}} dx$ |
| 2. $\int_1^{\infty} e^{-x} dx$        | 6. $\int_1^{\infty} \frac{1}{\sqrt{x}} dx$  | 10. $\int_1^{\infty} \frac{1}{\ln(x)} dx$        |
| 3. $\int_1^{\infty} \frac{1}{x^2} dx$ | 7. $\int_1^{\infty} e^{-x^2} dx$            | 11. $\int_1^{\infty} \frac{1}{x \ln(x)} dx$      |
| 4. $\int_1^{\infty} \sin(x) dx$       | 8. $\int_1^{\infty} \frac{1}{x^{1.003}} dx$ | 12. $\int_1^{\infty} \frac{1}{x \ln^2(x)} dx$    |

## Basic Rules for Convergence:

1. If  $\int f$  is convergent, then  $\int Kf$  is convergent for any constant  $K$ .
2. If  $\int f$  and  $\int g$  are convergent, then  $\int(f + g)$  is convergent.
3. If  $\int f$  is convergent and  $0 \leq g(x) \leq f(x)$  for all  $x$ , then  $\int g$  is convergent.

## Comparison Test

Find an appropriate “model function” to compare each of the following functions to. Decide whether each integral is convergent or divergent.

- |  |  |   |
|--|--|---|
| 1. $\int_1^{\infty} \frac{1}{(2x+1)^3} dx$ | 3. $\int_1^{\infty} \frac{x^2}{3+x^3} dx$    | 5. $\int_1^{\infty} \frac{1}{x+3\ln(x)} dx$ |
| 2. $\int_1^{\infty} \frac{1}{2x^2-x} dx$   | 4. $\int_1^{\infty} \frac{\sin^2 x}{x^2} dx$ | 6. $\int_1^{\infty} \frac{1}{x^2-3x+2} dx$  |

7.  $\int_1^{\infty} \frac{3 + \sin(x)}{e^x} dx$

8.  $\int_1^{\infty} \frac{1 + \ln(x)}{x} dx$

9.  $\int_1^{\infty} \frac{x^2 + 3x + 2}{x^4 - x + 1} dx$

### Singularities

Decide whether each of the following integrals are convergent or divergent:

1.  $\int_0^1 \frac{1}{x} dx$

4.  $\int_2^4 \frac{1}{x-3} dx$

7.  $\int_0^1 \frac{100000}{\sqrt{x}} dx$

2.  $\int_0^1 \frac{1}{x^2} dx$

5.  $\int_4^6 \frac{1}{x-3} dx$

8.  $\int_5^6 \frac{1}{(x-3)\sqrt{x-5}} dx$

3.  $\int_0^1 \frac{1}{\sqrt{x}} dx$

6.  $\int_1^2 \frac{1}{x^2-1} dx$

9.  $\int_0^1 x^2 dx$

### More Comparison Test

Pretty much the only function you care to compare things to here is  $1/x$  (or, more generally,  $1/x^p$ ).

1.  $\int_0^3 \frac{x}{x-2} dx$

5.  $\int_1^2 \frac{3x}{x-1} dx$

2.  $\int_6^1 \frac{(x-4)(3x+1)}{\sqrt{x-6}} dx$

6.  $\int_4^5 \frac{x + \ln(x) + 5\sqrt{x}}{\sqrt{x-4}} dx$

3.  $\int_{-1}^1 \frac{3x+5}{x^2+2x+1} dx$

7.  $\int_0^1 \frac{3x+1}{e^x} dx$

4.  $\int_0^1 \frac{x^2-x+3}{x^2-x} dx$

8.  $\int_{-2}^2 \frac{\sqrt{x} + 2e^{-x} - e^x}{x^{2/3}} dx$

### Harder Limits

1.  $\int_0^1 \frac{\sin(x)}{x} dx$

4.  $\int_0^1 \frac{e^{1/x}}{x^3} dx$

2.  $\int_0^1 x \ln(x) dx$

5.  $\int_0^1 \frac{\ln(1+x)}{x} dx$

3.  $\int_{-1}^0 \frac{e^{1/x}}{x^3} dx$

6.  $\int_0^1 \frac{\arctan x}{x} dx$