9.1: Modeling with Differential Equations

Monday, April 6

Warmup: The Marathon Runner

A runner running a marathon reaches the 6-mile marker after one hour, the 10-mile marker after 2 hours, and the 16-mile marker after 3 hours. Estimate the time it takes the runner to reach the finish line (26.2 miles).

One possible model for the runner's progress is $x(t) = \frac{2}{3}t^3 - 3t^2 + \frac{25}{3}t$. Why is this not a good model?

Gravity

Suppose we drop a bowling ball from the top of the Campanile (94 meters). Give a qualitative description of the ball's motion over time.

How is a good model for the bowling ball's motion related to the law of gravity?

How will things change if we drop a pillow instead?

Coffee

A cup of coffee is sitting in a large room.

1. Give a qualitative description of the change in the temperature of the coffee over time.

2. Can you find a model that has these same qualitative properties?

3. Suppose the coffee starts out colder than the surrounding air... what does your model suggest will happen?

Pendulum

A pendulum suspended by a metal rod is swinging back and forth.

1. When does gravity have the strongest effect on the pendulum's motion? When does it have a weak effect?

2. A model for the pendulum's motion is given by $\theta'' = -k\sin(\theta)$, where k is some positive constant. There are two functions $\theta(t)$ where the pendulum does not move at all over time–find both of them.

3. What is the difference between these two solutions?

Spread of a Rumor

Someone heard tell that Jar Jar Binks will have the starring role in the new Star Wars movie, and the exciting rumor spread like wildfire. Make the following assumptions about the spread of the rumor:

1. If a person who knows the rumor meets a person who does not know the rumor, they will spread the rumor.

2. Interactions between any pair of people are equally likely (that is, people do not have social cliques but are capable of bumping into anyone else with equal probability).

When should the rumor spread the fastest? Under what conditions will it not spread at all?
If $P(t)$ is the proportion of people (between 0 and 1) who know the rumor at time t, check that the model $P' = kP(1-P)$ is a reasonable one. Why would the models $P' = P$ or $P' = 1 - P$ not be reasonable?