11.6: Convergence Tests

Friday, March 6

Speed Round

1. $\cos(\pi n) =$
2. $\dfrac{2^{n+1}}{2^n} =$
3. $\dfrac{(n+1)!}{n!} =$
4. $\dfrac{(2n+2)!}{2n!} =$
5. $\dfrac{2^{n+1}}{n!}\dfrac{(n+1)!}{2^n} =$
6. $\lim\limits_{n\to\infty} \dfrac{(n+1)^2}{n^2} =$
7. $\lim\limits_{n\to\infty} n^{1/n} =$
8. $\lim\limits_{n\to\infty} e^{1/n} =$
9. $\lim\limits_{n\to\infty} e^{-1/n} =$

Convergent or divergent?

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+\ln n}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+3n}{n^3}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{n+\sqrt{n^3+3}}{n^3}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 2^n}{n!}$
5. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$
7. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3+\pi^n+\sin n}{3^n}$
8. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+\sqrt{2}}$
9. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$

The Convergence Tests

1. Alternating Series: If
   (a) $b_n > 0$ for all n
   (b) b_n is a monotonically decreasing sequence ($b_n > b_{n+1}$)
   (c) $\lim_{n\to\infty} b_n = 0$,

   then $\sum_{n=1}^{\infty}(-1)^n b_n$ converges.
2. Ratio Test: If $L = \lim\limits_{n\to\infty} \dfrac{|a_{n+1}|}{|a_n|} < 1$, then $\sum_{n=1}^{\infty} a_n$ converges. If $L > 1$, the series diverges, and if $L = 1$ the test is inconclusive.
3. Root test: If $L = \lim\limits_{n\to\infty} |a_n|^{1/n} < 1$ then $\sum_{n=1}^{\infty} a_n$ converges. If $L > 1$, the series diverges, and if $L = 1$ the test is inconclusive.

The Ratio Test and Root Test Do Not Care For Polynomials

Let $a_n = n^K$, for any number K. Show that the ratio and root tests are both inconclusive.

Exercises

Decide whether the following series are absolutely convergent, conditionally convergent, or divergent.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 2^n}{3^n}$
5. $\displaystyle\sum_{n=1}^{\infty} \frac{n^{10} 5^n}{n!}$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3 + (-5)^n}{4^n}$
7. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n - \ln n}$
8. $\displaystyle\sum_{n=1}^{\infty} \frac{n + (-3)^n}{3^n}$
9. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^3 + 4\sqrt{n}}{2n^3 + 1} \right)^n$
10. $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$
11. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (n + 2^n)}{n 2^n}$
12. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

Our Favorite Power Series

For each of the following power series, find the values of x for which the series is divergent, the values for which it is absolutely convergent, and the values (if any) for which it is conditionally convergent.

1. $e^x = 1 + x + x^2/2 + x^3/3! + \ldots = \displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n!}$

2. $\sin(x) = x - x^3/3! + x^5/5! - \ldots = \displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!}$

3. $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots = \displaystyle\sum_{n=0}^{\infty} x^n$

4. $\ln(1+x) = x - x^2/2 + x^3/3 - x^4/4 + \ldots = \displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$

5. $\arctan(x) = x - x^3/3 + x^5/5 - \ldots = \displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{2n-1}$