

11.5-11.6: Review

Monday, March 9

Speed Round

Evaluate the expressions, or determine whether the series converge or diverge:

1. $\cos(\pi n) =$

2. $e^x = 1 +$

3. $\sin(x) = x -$

4. $\frac{(n+2)!}{n!} =$

5. $\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n =$

6. $\frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n} =$

7. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

8. $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

9. $\sum_{n=1}^{\infty} \frac{(\ln n)^3}{n^2}$

10. $\sum_{n=1}^{\infty} \frac{1 + \sqrt{n}}{n^2}$

11. $\sum_{n=1}^{\infty} \frac{1 + \sqrt{n}}{n\sqrt{n}}$

12. $\sum_{n=1}^{\infty} \frac{2^n \cdot n^{100}}{n!}$

13. $\sum_{n=1}^{\infty} \frac{(-\pi)^n}{3^n}$

14. $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$

15. $\sum_{n=1}^{\infty} \frac{n^2 + 2^n}{3^n}$

Alternating Series

Draw a picture illustrating the proof of the Alternating Series test:

Which of the following series does the Alternating Series test apply to?

1. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

3. $\sum_{n=1}^{\infty} \frac{(-1)^n}{1 + \ln n}$

5. $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{1 + \ln n}$

2. $\sum_{n=1}^{\infty} \frac{1}{n}$

4. $\sum_{n=1}^{\infty} \frac{\cos n}{n}$

6. $\sum_{n=1}^{\infty} \frac{1 + 2(-1)^n}{n}$

Absolute Convergence

Find three different series that are convergent but not absolutely convergent. What do they have in common?

Various Tests

Decide whether the following series are absolutely convergent, conditionally convergent, or divergent. Decide which test is most appropriate.

1.
$$\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$$

5.
$$\sum_{n=1}^{\infty} \frac{n^2 \cdot 3^n}{n!}$$

9.
$$\sum_{n=1}^{\infty} (-1)^n (\sqrt{n+1} - \sqrt{n})$$

2.
$$\sum_{n=1}^{\infty} \frac{3^n}{n!}$$

6.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

10.
$$\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{1 + \ln n}$$

3.
$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

7.
$$\sum_{n=1}^{\infty} \frac{3 + (-1)^n}{n^{3/2}}$$

11.
$$\sum_{n=1}^{\infty} \frac{2^n \cdot n!}{(n+3)!}$$

4.
$$\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)(n!)}$$

8.
$$\sum_{n=1}^{\infty} \left(\frac{1 + 2 \ln n}{3 \ln n} \right)^n$$

12.
$$\sum_{n=1}^{\infty} (-1)^n \sin(1/n)$$

Some Assembly Required

Decide whether the following series are absolutely convergent, conditionally convergent, or divergent.

1.
$$\sum_{n=1}^{\infty} \left(1 - \frac{1}{n} \right)^{n^2}$$

2.
$$\sum_{n=1}^{\infty} (-1)^n (\sqrt{n+1} - \sqrt{n})$$

3.
$$\sum_{n=1}^{\infty} (-1)^n \sqrt{n} (\ln(n+1) - \ln n)$$

4.
$$\sum_{n=1}^{\infty} \frac{n^n}{n!}$$

5.
$$\sum_{n=1}^{\infty} (-1)^n n \tan(\pi/n)$$

6.
$$\sum_{n=1}^{\infty} \sin \frac{\pi}{n^2} - \sin \frac{\pi}{n^3}$$

Power Series

For what values of x will $\sum_{n=1}^{\infty} x^n/n$ converge?