9.1-9.2: Differential Equations and Direction Fields

1. Show that the function $y = x \sin x$ satisfies the differential equation $y'' + y = 2 \cos x$

2. Show that the function $y = xe^{-x} + 2$ satisfies the differential equation $y - xy' = x^2 e^{-x} + 2$

3. If $y' = y$ and $y(0) = 1$, estimate $y(1)$ using Euler's method with a step size of $\Delta x = 0.5$.

4. Match the following differential equations with the appropriate direction fields:

   (a) $y' = x + y - 1$
   (b) $y' = xy - 1$
   (c) $y' = x^2 + y^2 - 1$
   (d) $y' = x - y + 1$
   (e) $y' = x(y - 1)$
   (f) $y' = x^2 - y^2 + 1$
9.3: Separable Equations
1. \( y' = \frac{x}{y}, y(0) = -3 \)
2. \( y' = xy \sin x, y(0) = 1 \)
3. \( xy' - y = 1, y(2) = 3 \)

9.5: Linear Equations
1. \( xy' - y = 1, y(2) = 3 \)
2. \( 2xy' + y = 6x, y(4) = 20 \)
3. \( y' + xy = x, y(1) = 1. \)

17.1-2: Second-order Linear Equations
Solve each non-homogeneous equation using either variation of parameters or the method of undetermined coefficients, whichever is more appropriate.
1. \( y'' + 3y' + 2y = \sin x + 2 \cos x \)
2. \( y'' - 2y' + y = 2xe^x - e^x \)
3. \( y'' + y = 2 \sin x + 3 \)
4. \( y'' + y = \frac{1}{\cos x} \)
5. \( y'' + 3y' + 2y = \sin(e^x) \)