Review: Chapter 11

Friday, May 1

11.1: Sequences

True or False! As always, give a counterexample to the false statements.

1. If $\{a_n\}$and $\{b_n\}$are convergent then $\{a_n + b_n\}$is convergent.
2. If $\{a_n\}$and $\{b_n\}$are convergent then $\{a_n b_n\}$is convergent.
3. If $\{a_n\}$and $\{b_n\}$are divergent then $\{a_n + b_n\}$is divergent.
4. If $\{a_n\}$and $\{b_n\}$are divergent then $\{a_n b_n\}$is divergent.
5. If f is continuous and $\{a_n\}$converges then $\lim_{n\to\infty} f(a_n)$ exists.
6. If f is continuous and $\{a_n\}$diverges then $\lim_{n\to\infty} f(a_n)$ does not exist.

Not True or False!

1. Define what it means for a sequence to be bounded.
2. What are the conditions for the Monotone Convergence Theorem?
3. Give an example of a monotonic sequence that does not converge.
4. Give an example of a bounded sequence that does not converge.

11.2: Series

1. What is the harmonic series? Does it converge or diverge?
2. Decide whether $\displaystyle\sum_{n=1}^{\infty} 3 \cdot 2^n$ and $\displaystyle\sum_{n=1}^{\infty} 3/2^n$ converge or diverge. Find the limits if they converge.

11.3-11.7: Lots of Convergence Tests

Converge or diverge?

1. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^3+1}}{n^2}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$
5. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^{1.1}}$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{n^{30}}{1.01^n}$

For each of the following tests, do the following:

1. State what the test is.
2. Give an example of a series where the test proves convergence, if applicable.
3. Give an example of a series where the test proves divergence, if applicable.
4. Give an example of a series where the test is inconclusive or does not apply.

- Test For Divergence

- P-series Test
- Comparison Test
- Limit Comparison Test
- Alternating Series Test
- Ratio Test
- Root Test

True/False!

1. If $\sum_{n=1}^{\infty} a_n$ is convergent then it is absolutely convergent.
2. If $\sum_{n=1}^{\infty} a_n$ is absolutely convergent then it is convergent.
3. If the Ratio Test says that $\sum_{n=1}^{\infty} a_n$ converges then it converges absolutely.
4. If $\sum_{n=1}^{\infty} a_n$ converges but the Ratio Test is inconclusive then $\sum_{n=1}^{\infty} a_n$ converges conditionally.
5. If $\sum_{n=1}^{\infty} a_n$ is an alternating series then it converges.

11.8-10: Taylor Series

Find the Taylor series for the following functions up to the x^5 term:

1. $\sin x$
2. $\cos x$
3. e^x
4. $\frac{1}{1-x}$
5. $e^x \cos x$
6. $\frac{x^2}{1+2x}$

Find power series that have the following radii of convergence:

1. $[-1, 1]$
2. $(3, 5)$
3. $[2, 3)$
4. $(0, 7]$
5. $(-\infty, \infty)$
6. $\{4\}$