Review for Midterm 2

Convergence Tests

Limit Comparison Test

True/False: For each of the following claims, assume that a_n and b_n are sequences with positive terms.

1. If $\lim_{n\to\infty} a_n/b_n = 0$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
2. If $\lim_{n\to\infty} a_n/b_n = 0$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.
3. If $\lim_{n\to\infty} a_n/b_n = \infty$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
4. If $\lim_{n\to\infty} a_n/b_n = \infty$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.
5. If $\sum_{n=1}^{\infty} a_n$ converges, then $\ln(1 + a_n)$ converges.

Decide whether each of the following series converge or diverge. Give an appropriate sequence b_n to use with the Limit Comparison test.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+3)(n+5)}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+n}}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+3}}{(n+1)^2}$

Conditional Convergence

State the three conditions for the Alternating Series test:

List three different series that converge conditionally but not absolutely:

Decide whether the Alternating Series test applies to each of the following series:

1. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{1+3\cdot(-1)^n}{1+\ln(n)}$

Absolute Convergence

True/False:

1. If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
2. If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} \sin(n^2) a_n$ converges.

Show that the following series converge:

1. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$

Ratio Test/Radius of Convergence

True/False:

1. If a_n is any polynomial, then $\lim_{n\to\infty} |a_{n+1}|/|a_n| = 1$.
2. If the Ratio Test suggests that $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges absolutely.
3. If a_n is any polynomial, then the radius of convergence of $\sum_{n=1}^{\infty} a_n x^n$ is 1.
4. If the radius of convergence of $\sum_{n=1}^{\infty} a_n x^n$ is R, then $\lim_{n\to\infty} |a_{n+1}|/|a_n| = 1/R$.
5. If $\sum_{n=1}^{\infty} a_n x^n$ converges at $x = 1$ then it converges at $x = 1/2$.
6. If $\sum_{n=1}^{\infty} a_n/2^n$ converges then $\sum_{n=1}^{\infty} a_n \cdot 2^n$ converges.

Find the radius of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(5x-7)^n}{n^3}$.

Taylor Series

DO NOT TAKE DERIVATIVES UNLESS ABSOLUTELY NECESSARY

Find the first three non-zero terms of the Taylor series for $\cos(x)/(1+x)$ and $e^{2x}/(1+2x)$ at $x = 0$.