Review for Midterm 2

Convergence Tests

Limit Comparison Test

True/False: For each of the following claims, assume that a_n and b_n are sequences with positive terms.

1. If $\lim_{n\to\infty} a_n/b_n = 0$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. TRUE
2. If $\lim_{n\to\infty} a_n/b_n = 0$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges. FALSE
3. If $\lim_{n\to\infty} a_n/b_n = \infty$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. FALSE
4. If $\lim_{n\to\infty} a_n/b_n = \infty$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges. TRUE
5. If $\sum_{n=1}^{\infty} a_n$ converges, then $\ln(1 + a_n)$ converges. TRUE

Decide whether each of the following series converge or diverge. Give an appropriate sequence b_n to use with the Limit Comparison test.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+3)(n+5)}$: CONV: $b_n = 1/n^2$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+n}}$: DIV: $b_n = 1/n$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+3}}{(n+1)^2}$: CONV: $b_n = 1/n^{3/2}$

Conditional Convergence

State the three conditions for the Alternating Series test:

1. The series is strictly alternating.
2. $\lim_{n\to\infty} a_n = 0$
3. $|a_{n+1}| \leq |a_n|$

List three different series that converge conditionally but not absolutely:
$\sum_{n=1}^{\infty} (-1)^n/n, \sum_{n=1}^{\infty} (-1)^n/\sqrt{n}, \sum_{n=2}^{\infty} (-1)^n/\ln n$
Decide whether the Alternating Series test applies to each of the following series:

1. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n}$ NO: NOT STRICTLY ALTERNATING
2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ YES
3. $\displaystyle\sum_{n=1}^{\infty} \frac{1+3\cdot(-1)^n}{1+\ln(n)}$ NO: TERMS ARE NOT ALWAYS GETTING SMALLER

Absolute Convergence

True/False:

1. If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. TRUE
2. If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} \sin(n^2) a_n$ converges. TRUE

Show that the following series converge:

1. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$: COMPARE TO $1/n^2$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$: COMPARE TO $1/n\sqrt{n}$

Ratio Test/Radius of Convergence

True/False:

1. If a_n is any polynomial, then $\lim_{n\to\infty} |a_{n+1}|/|a_n| = 1$. TRUE
2. If the Ratio Test suggests that $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges absolutely. TRUE
3. If a_n is any polynomial, then the radius of convergence of $\sum_{n=1}^{\infty} a_n x^n$ is 1. TRUE
4. If the radius of convergence of $\sum_{n=1}^{\infty} a_n x^n$ is R, then $\lim_{n\to\infty} |a_{n+1}|/|a_n| = 1/R$. TRUE
5. If $\sum_{n=1}^{\infty} a_n x^n$ converges at $x = 1$ then it converges at $x = 1/2$. TRUE
6. If $\sum_{n=1}^{\infty} a_n/2^n$ converges then $\sum_{n=1}^{\infty} a_n \cdot 2^n$ converges. FALSE

Find the interval of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(5x-7)^n}{n^3}$.

INTERVAL: $[6/5, 8/5]$

Taylor Series

DO NOT TAKE DERIVATIVES UNLESS ABSOLUTELY NECESSARY

Find the first three non-zero terms of the Taylor series for $\cos(x)/(1+x)$ and $e^{2x}/(1+2x)$ at $x = 0$.

$$\frac{\cos x}{1+x} = 1 - x + x^2/2 - x^3/2 + \dots$$
$$\frac{e^{2x}}{1+2x} = 1 + 2x^2 - \frac{8}{3}x^3 + 6x^4 - \dots$$