**Math 1B, Second Midterm Examination**
9:00-10:00pm, N.Reshetikhin, March 21, 2014

Student's Name:

TA's name:

Student's i.d. number:

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|---|---|---|---|---|---|---|
| *Problem* | 1 | 2 | 3 | 4 | 5 | *Total* |
| *Points* | 24 | 20 | 20 | 21 | 15 | 100 |
| *Grade* | | | | | | |

1.(*24 points*) For each of the following series determine whether the series is divergent, conditionally convergent, or absolutely convergent. Indicate which tests you used.

a) (*8 points*)

$$\sum_{n=1}^{\infty}(-1)^n \frac{1}{(n^2+1)^{1/3}}$$

b) (*8 points*)

$$\sum_{n=1}^{\infty}\sin(n)\sqrt{\frac{n^2+n}{n^2+1}}$$

c) (*8 points*)

$$\sum_{n=6}^{\infty}\frac{\sin(n^2+2)}{n(n-5)}$$

2.(*20 points*)These are True-False questions. If the answer is True, you should explain why (concisely). If the answer is False, you should give a counter-example.

a) (*5 points*) The series $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} \sin(\ln(n))a_n$ converges.

b) (*5 points*) The sequence $\{b_n\}$ is divergent. The sequence $\{a_n\}$ converges, then the sequence $\{a_n b_n\}_{n=1}^{\infty}$ diverges.

c) (*5 points*) The series $\sum_{n=1}^{\infty} a_n$ converges absolutely and the sequence $\{b_n\}$ is bounded, then the series $\sum_{n=1}^{\infty} a_n b_n$ converges.

d) (*5 points*) The series $\sum_{n=1}^{\infty} a_n$ converges, then the series $\sum_{n=1}^{\infty} a_n^2$ converges.

3.(*20 points*)Find the first three non-zero terms of the the Taylor series about $x = 0$ for

$$f(x) = (1 + x)\ln(1 + x^2)$$

4.(*21 points*)Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty}(-1)^n \frac{(2x-1)^n}{n}$$

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5.(*15 points*)Answer True or False. You do not have to show your work.

a) (*3 points*) If $\sum_{n=1}^{\infty} a_n(x-1)^n$ converges at $x = 4$ and diverges at $x = -2$, then it converges at $x = -1$.

b) (*3 points*) If a series $\sum_{n=1}^{\infty} a_n 3^n$ converges, then $\sum_{n=1} a_n 2^n$ converges.

c) (*3 points*) If the series $\sum_{n=1}^{\infty} a_n$ converges conditionally, then the radius of convergence of $\sum_{n=1}^{\infty} a_n(x-1)^n$ is 1.

d) (*3 points*) It is possible that the series $\sum_{n=1}^{\infty} a_n 3^n$ converges absolutely, but the series $\sum_{n=1}^{\infty} a_n(-2)^n$ diverges.

e) (*3 points*) If the series $\sum_{n=1}^{\infty} a_n x^n$ has radius of convergence 1, then $\sum_{n=1}^{\infty} \frac{a_n}{n^3}$ converges.