1. (24 points) For each of the following series determine whether the series is divergent, conditionally convergent, or absolutely convergent. Indicate which tests you used.

a) (8 points)
\[ \sum_{n=1}^{\infty} (-1)^n \frac{1}{(n^2 + 1)^{1/3}} \]

b) (8 points)
\[ \sum_{n=1}^{\infty} \sin(n) \sqrt{\frac{n^2 + n}{n^2 + 1}} \]

c) (8 points)
\[ \sum_{n=6}^{\infty} \frac{\sin(n^2 + 2)}{n(n - 5)} \]
2. (20 points) These are True-False questions. If the answer is True, you should explain why (concisely). If the answer is False, you should give a counter-example.

a) (5 points) The series \( \sum_{n=1}^{\infty} a_n \) converges absolutely, then \( \sum_{n=1}^{\infty} \sin(\ln(n))a_n \) converges.

b) (5 points) The sequence \( \{b_n\} \) is divergent. The sequence \( \{a_n\} \) converges, then the sequence \( \{a_nb_n\}_{n=1}^{\infty} \) diverges.

c) (5 points) The series \( \sum_{n=1}^{\infty} a_n \) converges absolutely and the sequence \( \{b_n\} \) is bounded, then the series \( \sum_{n=1}^{\infty} a_nb_n \) converges.

d) (5 points) The series \( \sum_{n=1}^{\infty} a_n \) converges, then the series \( \sum_{n=1}^{\infty} a_n^2 \) converges.
3. (20 points) Find the first three non-zero terms of the Taylor series about $x = 0$ for

$$f(x) = (1 + x) \ln(1 + x^2)$$
4. (21 points) Find the interval of convergence for the power series

\[ \sum_{n=1}^{\infty} \frac{(-1)^n (2x - 1)^n}{n} \]
5. (15 points) Answer True or False. You do not have to show your work.

a) (3 points) If \( \sum_{n=1}^{\infty} a_n (x - 1)^n \) converges at \( x = 4 \) and diverges at \( x = -2 \), then it converges at \( x = -1 \).

b) (3 points) If a series \( \sum_{n=1}^{\infty} a_n 3^n \) converges, then \( \sum_{n=1}^{\infty} a_n 2^n \) converges.

c) (3 points) If the series \( \sum_{n=1}^{\infty} a_n \) converges conditionally, then the radius of convergence of \( \sum_{n=1}^{\infty} a_n (x - 1)^n \) is 1.

d) (3 points) It is possible that the series \( \sum_{n=1}^{\infty} a_n 3^n \) converges absolutely, but the series \( \sum_{n=1}^{\infty} a_n (-2)^n \) diverges.

e) (3 points) If the series \( \sum_{n=1}^{\infty} a_n x^n \) has radius of convergence 1, then \( \sum_{n=1}^{\infty} \frac{a_n}{n^n} \) converges.