1. 15 pts Evaluate the integral

\[ \int \sin(2x)e^x \, dx \]
2.15 \textit{pts} Compute the integral
\[
\int \frac{1}{(t - 1)(t^2 + 1)} \, dt
\]
3.15 pts Indicate which of the following statements are true and which are false. Do not show your work. Each correct answer is worth 3 pts and each wrong answer is worth 0 pts.

1. \( \int_{1}^{\infty} \frac{dx}{x \ln x} \) is a convergent improper integral.

2. \( \int_{1}^{\infty} \frac{1}{x(\ln x)^2} \) is a convergent improper integral.

3. \( \int_{\frac{\pi}{2}}^{\pi} \frac{(\cos x)^2}{x - \frac{\pi}{2}} \) is a divergent improper integral.

4. \( \int_{0}^{\infty} \frac{\sin(x^2)}{x^2} \) is a convergent improper integral.

5. \( \int_{0}^{1} \frac{dx}{x \sqrt{1 - x}} \) is a convergent improper integral.
4.15 pts Find the radius and the interval of convergence of the power series

\[
\sum_{n=0}^{\infty} (-1)^n \frac{1}{\sqrt{n+5}} (x-2)^n
\]
5.15 pts State whether each of the following series is absolutely convergent, conditionally convergent, or divergent. You do not have to show your work. Each correct answer is worth 3 pts and each wrong answer is worth 0 pts.

1. \[ \sum_{n=1}^{\infty} (-1)^n \sqrt[n+1]{} - \sqrt[n]{} \]

2. \[ \sum_{n=1}^{\infty} \cos\left(\frac{\pi i}{2} + \frac{1}{n^2}\right). \]

3. \[ \sum_{n=1}^{\infty} \frac{n \sin n}{n^3 + 1}. \]

4. \[ \sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right) \sin\left(\frac{\pi}{2} + \pi n\right). \]

5. \[ \sum_{n=1}^{\infty} n^2 \cos\left(\frac{\pi n}{2}\right). \]
6.15 pts These are True-False questions. If the answer is True, you should explain why (concisely). If the answer is False, you should give a counter-example. Each problem is worth 5 points if the answer is correct and 0 points if the answer is not correct.

1. If the series \( \sum_{n=1}^{\infty} a_n \) converges and the series \( \sum_{n=1}^{\infty} a_n^2 \) diverges, then \( \sum_{n=1}^{\infty} a_n \) also converges conditionally.

2. If the series \( \sum_{n=1}^{\infty} a_n \) converges, then the series \( \sum_{n=1}^{\infty} a_n + |a_n| \) converges.

3. If the series \( \sum_{n=1}^{\infty} a_n \) converges and the sequence \( \{b_n\}_{n=1}^{\infty} \) converges as \( n \to \infty \), and \( b_n \neq 0 \), then \( \sum_{n=1}^{\infty} a_n b_n \) converges.
For each statement indicate whether it is true or false. You do not have to show your work. Each correct answer is worth 3 points and each wrong answer is worth 0 points.

1. If a series \( \sum_{n=1}^{\infty} (-1)^n a_n \) diverges and \( R \) is the radius of convergence, then the radius of convergence of \( \sum_{n=0}^{\infty} a_n (x - 1)^n \), then \( R \leq 2 \).

2. If \( \sum_{n=1}^{\infty} c_n (x - 1)^n \) converges at \( x = 3 \), then \( \sum_{n=1}^{\infty} c_n \) converges.

3. The radius of convergence of \( \sum_{n=1}^{\infty} (1 + 5^n) x^n \) is greater than 4.

4. Even though the series \( \sum_{n=1}^{\infty} c_n (x - 1)^n \) converges at \( x = -1 \), the series \( \sum_{n=1}^{\infty} c_n 2^n \) may diverge.

5. The series \( \sum_{n=1}^{\infty} c_n x^n \) converges absolutely for \( |x| \leq 2 \) then the radius of convergence is 2.
8.15 pts Find the general solution to the differential equation

\[ yy' - y^2 x = x . \]
9.15 pts Find the solution to the initial value problem

\[ y'' + y = x^2 + e^x, \quad y(0) = -\frac{3}{2}, \quad y'(0) = \frac{1}{2}. \]
10.15 \textit{pnts} Find the solution to the initial value problem

\[ y' + \tan xy = \sec x, \quad y(0) = 0. \]
11.15 pnts Match pictures to differential equations.

a. \( \frac{dy}{dx} = y^3 - x^3 \)

b. \( \frac{dy}{dx} = \frac{y^2}{x^2} \)

c. \( \frac{dy}{dx} = -x + y \)

d. \( \frac{dy}{dx} = x^2 + y^2 \)

e. \( \frac{dy}{dx} = y^2 - x^2 \)
12.15 pts Find the power series solution to the initial value problem:

\[ xy'' + xy = 0, \quad y(0) = 1, \quad y'(0) = 0. \]