Math 1B, Final Examination

N.Reshetikhin, May 12, 2014

Problem	1	2	3	4	5	6	7	8	9	10	11	12	*Total*
Points	15	15	15	15	15	15	15	15	15	15	15	15	180
Grade													

Student's Name:

GSI's name:

Student's i.d. number:

1.*15 pnts* Evaluate the integral

$$\int \sin(2x)e^x dx$$

2. *15 pnts* Compute the integral

$$\int \frac{1}{(t-1)(t^2+1)} dt$$

3.*15 pnts* Indicate which of the following statements are true and which are false. Do not show your work. Each correct answer is worth *3 pnts* and each wrong answer is worth 0 pnts.

1. $\displaystyle\int_1^\infty \frac{dx}{x\ln x}$ is a convergent improper integral.

2. $\displaystyle\int_1^\infty \frac{1}{x(\ln x)^2}dx$ is a convergent improper integral.

3. $\displaystyle\int_{\frac{pi}{2}}^\infty \frac{(\cos x)^2}{x-\frac{\pi}{2}}dx$ is a divergent improper integral.

4. $\displaystyle\int_0^\infty \frac{\sin(x^2)}{x^2}dx$ is a convergent improper integral.

5. $\displaystyle\int_0^1 \frac{dx}{x\sqrt{1-x}}$ is a convergent improper integral.

4. *15 pnts* Find the radius and the interval of convergence of the power series

$$\sum_{n=0}^{\infty}(-1)^n \frac{1}{\sqrt{n+5}}\ (x-2)^n$$

5. *15 pnts* State whether each of the following series is absolutely convergent, conditionally convergent, or divergent. You do not have to show your work. Each correct answer is worth *3 pnts* and each wrong answer is worth 0 pnts.

1. $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n+1} + \sqrt{n}}$.

2. $\sum_{n=1}^{\infty} \cos(\frac{pi}{2} + \frac{1}{n^2})$.

3. $\sum_{n=1}^{\infty} \frac{n \sin n}{n^3 + 1}$.

4. $\sum_{n=1}^{\infty} \tan(\frac{1}{n}) \sin(\frac{\pi}{2} + \pi n)$.

5. $\sum_{n=1}^{\infty} n^2 \cos(\frac{\pi n}{2})$

6. *15 pnts* These are True-False questions. If the answer is True, you should explain why (concisely). If the answer is False, you should give a counter-example. Each problem is worth 5 points if the answer is correct and 0 points if the answer is not correct.

1. If the series $\sum_{n=1}^{\infty} a_n$ converges and the series $\sum_{n=1}^{\infty} a_n^2$ diverges, then $\sum_{n=1}^{\infty} a_n$ also converges conditionally.

2. If the series $\sum_{n=1}^{\infty} a_n$ converges, then the series $\sum_{n=1}^{\infty} a_n + |a_n|$ converges.

3. If the series $\sum_{n=1}^{\infty} a_n$ converges and the sequence $\{b_n\}_{n=1}^{\infty}$ converges as $n \to \infty$, and $b_n \neq 0$, then $\sum_{n=1}^{\infty} a_n b_n$ converges.

7. *15 pnts* For each statement indicate whether it is true or false. You do not have to show your work. Each correct answer is worth *3 pnts* and each wrong answer is worth 0 pnts.

1. If a series $\sum_{n=1}^{\infty}(-1)^n a_n$ diverges and R is the radius of convergence, then the radius of convergence of $\sum_{n=0}^{\infty} a_n(x-1)^n$, then $R \leq 2$.

2. If $\sum_{n=1}^{\infty} c_n(x-1)^n$ converges at $x = 3$, then $\sum_{n=1}^{\infty} c_n$ converges.

3. The radius of convergence of $\sum_{n=1}^{\infty}(1+5^n)x^n$ is greater 4.

4. Even though the series $\sum_{n=1}^{\infty} c_n(x-1)^n$ converges at $x = -1$, the series $\sum_{n=1}^{\infty} c_n 2^n$ may diverge.

5. The series $\sum_{n=1}^{\infty} c_n x^n$ converges absolutely for $|x| \leq 2$ then the radius of convergence is 2.

8. *15 pnts* Find the general solution to the differential equation

$$yy' - y^2 x = x \ .$$

9. *15 pnts* Find the solution to the initial value problem

$$y'' + y = x^2 + e^x, \quad y(0) = -\frac{3}{2}, \ \ y'(0) = \frac{1}{2}\,.$$

10.*15 pnts* Find the solution to the initial value problem

$$y' + \tan x y = \sec x, \quad y(0) = 0 \ .$$

11.*15 pnts* Match pictures to differential equations.

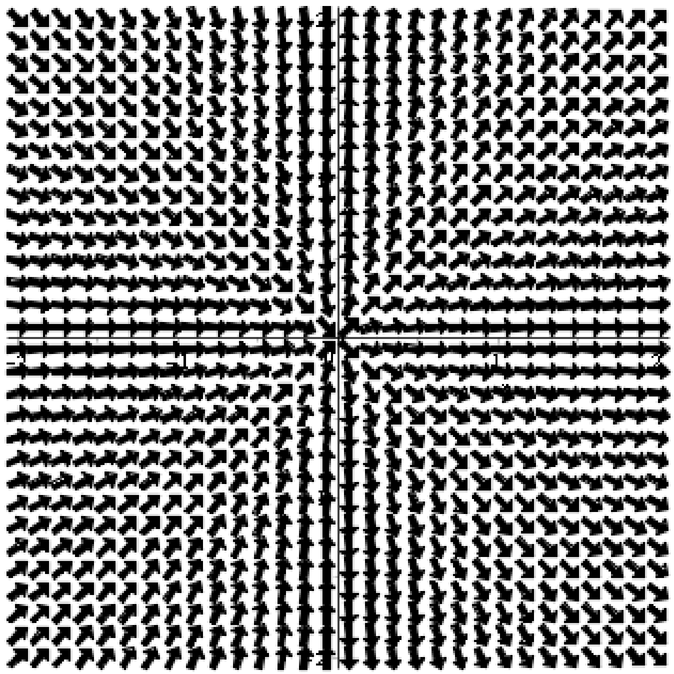
a. $\dfrac{dy}{dx} = y^3 - x^3$

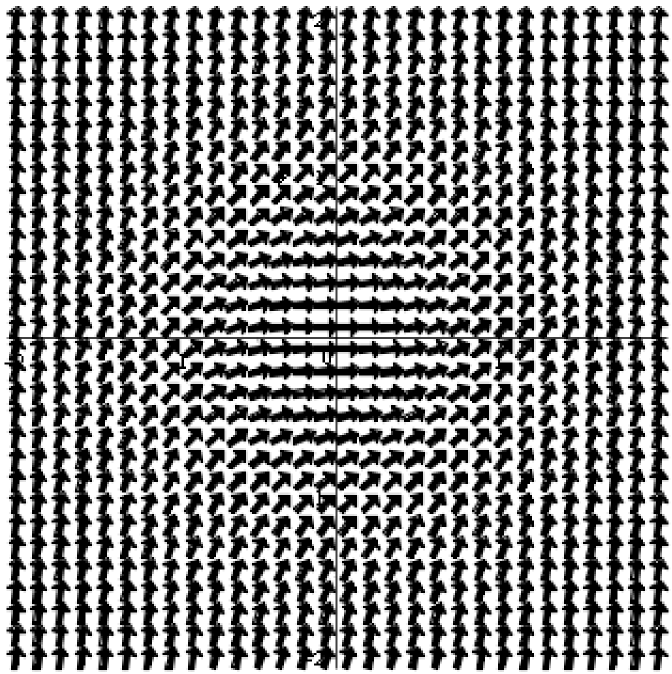
b. $\dfrac{dy}{dx} = \dfrac{y^2}{x^2}$

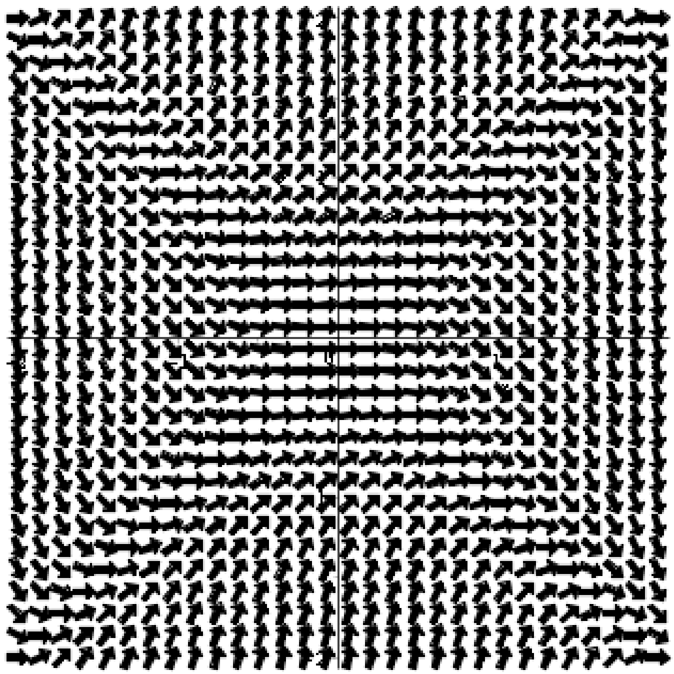
c. $\dfrac{dy}{dx} = -x + y$

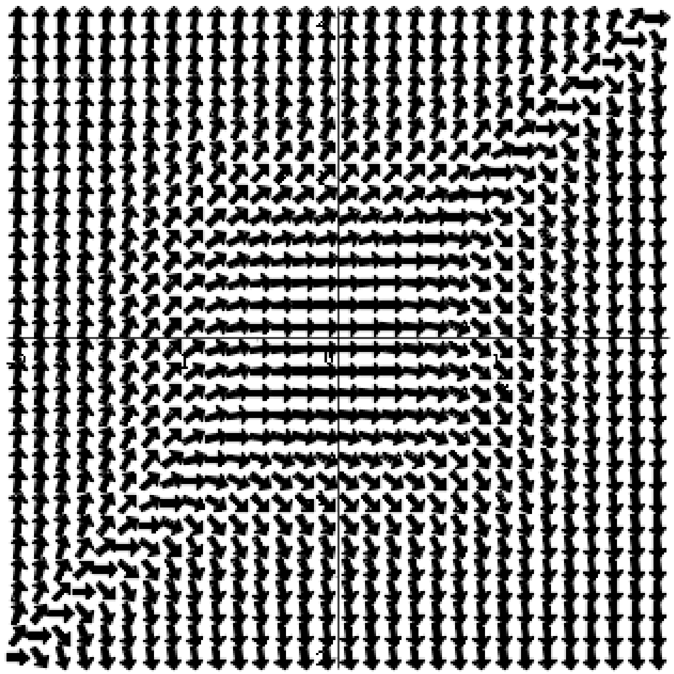
d. $\dfrac{dy}{dx} = x^2 + y^2$

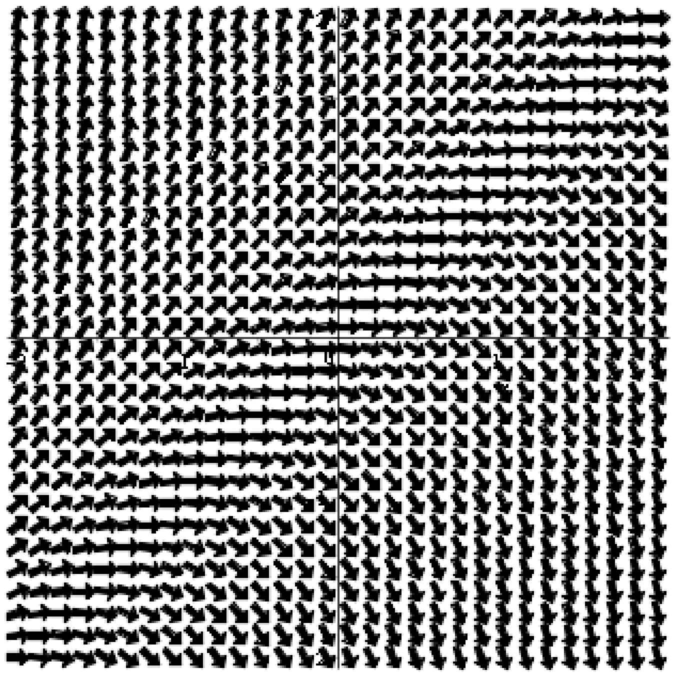
e. $\dfrac{dy}{dx} = y^2 - x^2$











12.*15 pnts* Find the power series solution to the initial value problem:

$$xy'' + xy = 0, \quad y(0) = 1, \quad y'(0) = 0 \ .$$