6.1-6.2: Orthogonality and Projection

Tuesday, October 11

Warmup

Define $\mathbf{u} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 & 2 & -1 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}$. Find the following:

1. $\|\mathbf{u}\|$
2. $\|\mathbf{v}\|$
3. $\|\mathbf{w}\|$
4. $\|\mathbf{u} + 2\mathbf{v}\|^2$
5. $\|\mathbf{u}\|^2 + 4\|\mathbf{v}\|^2$
6. $\|-3\mathbf{u}\|$
7. $\|\mathbf{u} + \mathbf{w}\|^2$
8. $\|\mathbf{v} + \mathbf{w}\|^2$
9. $\mathbf{v} \cdot (\mathbf{u} - 2\mathbf{w})$
10. $\mathbf{v} \cdot \mathbf{u} - 2\mathbf{v} \cdot \mathbf{w}$

Describe the shortest path from a point to a line.

Orthogonal Complements

If $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 1 \\ 0 & -3 \end{bmatrix}$, find a basis for $(Col(A))^{\perp}$.

Projections

Define $\mathbf{b}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$, and let $\hat{\mathbf{y}}$ be the projection of $\mathbf{y}$ onto the span of $\mathbf{b}_1$.

1. Verify that $\mathbf{b}_1$ and $\mathbf{b}_2$ are orthogonal.
2. If $\mathbf{y} = c_1\mathbf{b}_1 + c_2\mathbf{b}_2$, find an expression for $\hat{\mathbf{y}}$ in terms of the c_i and $\mathbf{b}_i$.
3. What are c_1 and c_2 in terms of $\mathbf{b}_1 \cdot \mathbf{y}$ and $\mathbf{b}_2 \cdot \mathbf{y}$?
4. Sketch a triangle whose vertices are the origin, $\mathbf{y}$, and $\hat{\mathbf{y}}$. Label the lengths of the sides, and find an expression for $\cos(\mathbf{y}, \mathbf{b}_1)$. Label the appropriate angle.
5. Find c_1 and c_2.
6. Show that $\mathbf{y} - \hat{\mathbf{y}}$ and $\hat{\mathbf{y}}$ are orthogonal.

Orthogonal Matrices

Find a matrix P such that $P \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_1 \\ x_2 \end{bmatrix}$. Find expressions for $\|\mathbf{x}\|^2$ and $\|P\mathbf{x}\|^2$. Is P orthogonal?