6.3-6.4: Projection and Gram-Schmidt

Thursday, October 13

Projection

Find the point on the line that passes through (0,0) and (3,1) closest to the point (5,5). Draw a picture.

If Π is the matrix that projects a point onto the line mentioned above, find Π explicitly. What is its eigendecomposition?

Gram-Schmidt

Find an orthogonal basis for $\text{Span}\left\{\begin{bmatrix}0\\4\\2\end{bmatrix}, \begin{bmatrix}5\\6\\-7\end{bmatrix}\right\}$.

True/False

1. The sum of two orthogonal matrices is an orthogonal matrix.
2. The product of two orthogonal matrices is an orthogonal matrix.
3. If U is orthogonal then $\|U\mathbf{x}\| = \|\mathbf{x}\|$ for any $\mathbf{x}$.
4. If U is orthogonal the the angle between $U\mathbf{x}$ and $U\mathbf{y}$ is the same as the angle between $\mathbf{x}$ and $\mathbf{y}$ for any $\mathbf{x}$ and $\mathbf{y}$.
5. If A and B are both diagonalizable and have the same eigenvectors then $AB = BA$.
6. If $A^2 = A$ then all eigenvalues of A are 0 or 1.
7. If all eigenvalues of A are 0 or 1 then $A^2 = A$.
8. For all $\mathbf{y}$ and each subspace W the vector $\mathbf{y} - \text{proj}_W \mathbf{y}$ is orthogonal to W.
9. For any $\mathbf{y} \in \mathbb{R}^n$ and any subspace $W \subset \mathbb{R}^n$, $\text{proj}_W(\text{proj}_W \mathbf{y}) = \text{proj}_W \mathbf{y}$.