5.2-5.3: Eigenvalues and Diagonalization

Tuesday, October 4

Warmup

True/False:

1. Eigenvectors with distinct eigenvalues are linearly independent.
2. The set of eigenvectors of a matrix A with a particular eigenvalue λ forms a subspace.
3. If A and B are row equivalent then A and B have the same determinant.
4. If A and B are row equivalent then A and B have the same null space.

Find the determinant of $\begin{bmatrix} 3-\lambda & 2 \\ 0 & -\lambda \end{bmatrix}$ in terms of λ. When is it equal to zero?

If $V = [\mathbf{v}_1, \ldots, \mathbf{v}_n]$ and Λ is a diagonal matrix with $\{\lambda_1, \ldots, \lambda_n\}$ as its diagonal elements, what is $V\Lambda$?

Diagonalizing a Matrix

Let A be the matrix $\begin{bmatrix} -4 & -8 \\ 3 & 7 \end{bmatrix}$. Find the characteristic polynomial of A.

Find the eigenvalues of A, and find eigenvectors for those eigenvalues.

Verify that the eigenvectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent, and therefore that $V = [\mathbf{v}_1, \mathbf{v}_2]$ is invertible.

If $AV = V\Lambda$ and V is invertible, then $V^{-1}AV = \Lambda$. Verify that this is true.

Find A^4 in terms of Λ and V. Show that your formula is correct.

Use your previous answer to find A^4.

Complex Numbers and Eigenvalues

Let i be a number such that $i^2 = -1$. Simplify:

1. $i^{10} + 3i^3$
2. $(3 + i)(1 - 2i)$
3. $\overline{2 + i}$
4. $\frac{1}{1-i}$

If $A^2 = 0$, show that 0 is the only possible eigenvalue of A. Does A have to be the zero matrix?

If $A^2 = -I$, is it possible for A to have real eigenvalues?