2.3-2.5: Matrix Algebra

Tuesday, September 13

Logic Warmup

State the *contrapositive* (i.e. “if (not B) then (not A)”) of each of the following:

1. If mn is odd then m is odd and n is odd.
2. If a number n is divisible by 2 and 3 then it is divisible by 6.
3. If a function f is not one-to-one then for every function g, $g \circ f$ is also not one-to-one.

Block Matrices

Evaluate each of the following matrix products:

1. $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$

2. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 4 & 5 \end{bmatrix}$

Prove the following (assuming all relevant matrices are $n \times n$). For which statements is it simpler to prove the contrapositive?

1. If $A\mathbf{x} = \mathbf{0}$ has a unique solution then A is one-to-one.
2. A is invertible if and only if A^T is invertible.
3. If A is not invertible then BA is also not invertible.
4. If A is onto then A^2 is also onto.

Evaluate the matrix product $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 3 \\ 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ first by finding each entry as an inner product, then as a sum of outer products. Which is simpler?

If $A = \mathbf{u}\mathbf{v}^T$ where $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$, what is the span of the columns of A?

Evaluate the following:

1. $\begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} B \\ C \end{bmatrix}$
2. $\begin{bmatrix} A & B \\ 0 & I \end{bmatrix}^4$
3. $\begin{bmatrix} I & B \\ 0 & I \end{bmatrix}^{-1}$
4. $\begin{bmatrix} I & A & 0 \\ 0 & I & B \\ 0 & 0 & I \end{bmatrix}^{-1}$

If $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} L_1 & 0 \\ L_2 & I \end{bmatrix} \begin{bmatrix} U_1 & U_2 \\ 0 & S \end{bmatrix}$, find the following:

1. L_2 and U_2, in terms of L_1, U_1, and the blocks of A.
2. S, in terms of A_{22} and the blocks of L and U.
3. S, entirely in terms of the blocks of A.