4.1-4.2: Vector Spaces

Thursday, September 15

0.1 Matrix Patterns

Define $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Find DA, AD, PA, AP^T, and PAP^T.

Calculate the determinants of the following matrices

1. $\begin{bmatrix} 1 & 3 & -6 & \pi^\pi & 1 \\ 2 & \sqrt{2} & 77 & 0 & 2 \\ 3 & -3 & .3 & .03 & 3 \\ 4 & 10^{100} & \sin(12) & 8 & 4 \\ 5 & 0 & -5 & 0 & 5 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \end{bmatrix}$

3. $\begin{bmatrix} -4 & 0 & 1 & 0 \\ 0 & 3 & 3 & 2 \\ 1 & 0 & -1 & -1 \\ 2 & 6 & 0 & 0 \end{bmatrix}$

Subspaces

Describe, geometrically, *all* subspaces of $\mathbb{R}^3$.

Which of the following are subspaces of the space of 2×2 matrices? Justify your answers:

1. $\{A : A = -A^T\}$
2. $\{A : A^2 = I\}$

Derivative

Let V be the set of all infinitely differentiable functions on $\mathbb{R}$, and let $D : V \to V$ be the derivative operator (i.e. $D(f) = f'$).

1. Is D a linear tranformation? Justify your answer.
2. Which of the following are subspaces?
   (a) $\{f : D(f) = 0\}$
   (b) $\{f : D(f) = 1\}$
   (c) $\{f : D(f) = ax + b, a, b, \in \mathbb{R}\}$
3. What is the kernel of D?
4. Let $\mathbb{P}_2$ be the set of degree 2 polynomials, and represent the polynomial $a_2x^2 + a_1x + a_0$ $\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$. What is the matrix representation of D?
5. What would this look like if we tried it for the space of *all* polynomials?

If $A = \begin{bmatrix} 1 & 3 & -4 & -3 & 1 \\ 0 & 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, what are Col(A) and Nul(A)?

If A and B are matrices, what is the relation between Col(A) and Col(AB)? What about Nul(B) and Nul(AB)?