1.9,2.1: Linear Transformations, Matrix Algebra

Tuesday, September 6

More Logic

The *negation* of a statment is true if and only if that statement is false. For example, the negation of "All ravens are black" is "There exists a raven that is not black." Negate the following statements:

1. All roses are either red or white.
2. If a bird is black, then that bird is a raven.
3. There exist animals that have wings but cannot fly.
4. If x is positive then $x^2 - 3x + 1$ is also positive.
5. If $AB = 0$ then $A = 0$ or $B = 0$.

Bonus warmup question: what is $\sum_{i=1}^{2} \sum_{j=1}^{2} 2^{i-j}$?

Find negations for the following statements. Decide whether the statements are true or false and justify your answers.

1. If T is a linear transformation and T is one-to-one, then T is onto.
2. The function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$ is a linear transformation.
3. The function $g : \mathbb{R} \to \mathbb{R}$ given by $g(x) = e^x - 3$ is onto.

State the negations of the following statements, and find counterexamples with 2×2 matrices: "For all matrices A and B..."

- $AB = BA$. (Hint: try a shear and a reflection or rotation)
- If $A \neq 0$ and $AB = AC$, then $B = C$
- If $AB = 0$ then $A = 0$ or $B = 0$.

Linear Transformations

If $f(x) = mx + b$, for what values of m and b is f a linear transformation? When it is linear, express its standard matrix representation in terms of m and b.

If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, illustrate the effect of A on the standard basis vectors $\mathbf{e}_1$ and $\mathbf{e}_2$. Find A^2, A^3, and A^4, and describe the associate linear transformations.

If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, what is A^n? Describe the geometric effect of applying A to a vector repeatedly. Find a matrix B such that $AB = I$.

If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, find all possible combinations of products of A and B and illustrate their effects on the letter "R"