1.3-1.4: Vector Equations and $Ax = b$

Tuesday, August 30

Functions

Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = 2x$. What is $\{f(x) : x \in \mathbb{R}\}$? Does $f(x)$ have an inverse function? If so, is its inverse defined on all of $\mathbb{R}$?

Answer the same questions as above for $g(x) = x^2$ and $h(x) = e^x$.

Let's make a secret code with the encryption function E('a') = 'm', E('b') = 'm',..., E("z") = "m". Use it to encrypt the message "hello world". Is this a useful secret code?

If possible, find functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \{a, b, c\} \to \{1, 2, 3\}$ that are...

- 1-1 but not onto
- onto but not 1-1
- both
- neither

Vector Equations

Define $\mathbf{u} := \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \mathbf{v} := \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \mathbf{w} := \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ for the rest of this section. Sketch the following:

- $\mathbf{u} + \mathbf{v}$
- $\mathbf{v} + \mathbf{u}$
- $\frac{1}{2}\mathbf{w} - 2\mathbf{u}$
- $\mathbf{w} + 2\mathbf{v}$
- $\text{Span}(\mathbf{v}, \mathbf{w})$
- $\text{Span}(\mathbf{u}, \mathbf{v})$

If we define $\mathbf{b} := \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, is $\mathbf{b}$ in $\text{Span}(\mathbf{u}, \mathbf{v}, \mathbf{w})$? What about $\text{Span}(\mathbf{v}, \mathbf{w})$?

Find a vector $\mathbf{y}$ such that the system $c_1\mathbf{v} + c_2\mathbf{w} = \mathbf{y}$ has infinitely many solutions.

A robot begins at the point $(0, 0, 0)$ and is capable of moving in the directions $\pm(1, 1, 1)$ and $\pm(-1, 3, 0)$. Find a point in space that the robot cannot reach.

Matrix Equations

If we define $A := \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} \end{bmatrix}$, write $\mathbf{v} + \mathbf{u}$, $\frac{1}{2}\mathbf{w} - 2\mathbf{u}$, and $\mathbf{w} + 2\mathbf{v}$ as matrix-vector products.

Recall that if A is an m-by-n matrix then

- $A(\mathbf{v} + \mathbf{u}) = A\mathbf{v} + A\mathbf{u}$
- $A(c\mathbf{v}) = c \cdot A\mathbf{v}$

for any $\mathbf{v}, \mathbf{u} \in \mathbb{R}^n$ and any $c \in \mathbb{R}$. Using these two facts, prove that if $A\mathbf{v}_1 = \mathbf{w}_1$ and $A\mathbf{v}_2 = \mathbf{w}_2$ then the system $A\mathbf{x} = 3\mathbf{w}_1 - \mathbf{w}_2$ is consistent.