# 9.7-9.8: Nonhomogeneous systems, Matrix exponentials

Tuesday, November 22

## Variation of Parameters

$$\mathbf{x}(t) = \mathbf{X}(t)\mathbf{c} + \mathbf{X}(t)\int \mathbf{X}^{-1}(s)\mathbf{f}(s)\,ds$$

Use the method of variaton of parameters given above to find a particular solution of the system

$$\mathbf{x}'(t) = \begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 2e^t \\ 4e^t \end{bmatrix}.$$

Also solve the problem using the method of undetermined coefficients. Which is simpler?

Use the method of undetermined coefficients to find a particular solution to the system

$$\mathbf{x}'(t) = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} \sin t \\ \cos t + \sin t \end{bmatrix}.$$

## The Matrix Exponential: Recap

- $e^{At}$ is a fundamental matrix for the system $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$.
- If $X(t)$ is a fundamental matrix then $e^{At} = X(t)X(0)^{-1}$.
- If $A = V\Lambda V^{-1}$ then $e^{A} = Ve^{\Lambda}V^{-1}$.

Find $e^{At}$, where $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$.

If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, what are the eigenvectors of $A$ and $B$? Find formulas for $A^n$ and $B^n$. What are $e^A$ and $e^B$?