6, 9.4: Higher-Order Equations

Tuesda

Linear Operators and Linear Independence

Find a differential equation for which $\{1, \sin t, \cos t\}$ is a fundamental solution set. Use the Wronskian to demonstrate that these three functions are linearly independent.

Show that $(D-1)(D+2)[t^2-2t+5] = (D+2)(D-1)[t^2-2t+5]$.

If D is the derivative operator, find a general solution to the differential equation

$$(D-1)^2(D+3)(D^2+2D+5)^2[y] = 0$$

Show that the functions e^t, t^2, and $\sin t$ are linearly independent by finding for each term a linear operator that annihilates the other two terms but not that one. Why does this show that the functions are linearly independent?

Matrix Methods for Linear Systems

Write the given system in the matrix form $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$:

$$x'(t) = 3x(t) - y(t) + t^2$$
$$y'(t) = -x(t) + 2y(t) + e^t$$

Write the system $\mathbf{x}' = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} \mathbf{x} + e^t \begin{bmatrix} t \\ 1 \end{bmatrix}$ as a set of scalar equations.

Verify that the vector functions $\mathbf{x}_1 = \begin{bmatrix} e^t \\ e^t \end{bmatrix}$ and $\mathbf{x}_2 = \begin{bmatrix} e^{-t} \\ 3e^{-t} \end{bmatrix}$ are solutions to the system $\mathbf{x}' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \mathbf{x}$.