4.1-4.2: Linear Second-Order Equations

Tuesday, November 1

Second-order Linear Equations

Verify that the function $y(t) = e^{-3t}\sin(\sqrt{3}t)$ is a solution to the equation $my'' + by' + ky = 0$ if $m = 1, b = 6, k = 12$. What happens to the solution as $t \to \infty$?

Find the solution to the initial value problem $y'' - 4y' + 3y = 0$, $y(0) = 1$, $y'(0) = 1/3$.

Find the solution to the initial value problem $y'' - 4y' + 4y' = 0$, $y(1) = 1$, $y'(1) = 1$.

Linear Independence and Subspaces

If $C^2(\mathbb{R})$ is the vector space of twice-differentiable functions on $\mathbb{R}$, show that for fixed a, b, c the set of solutions to the differential equation $ay'' + by' + cy = 0$ is a subspace of $C^2(\mathbb{R})$. Better yet: show that the solution set is the null space of a particular linear transformation.

If f and g are linearly dependent functions, show that $(f/g)' = 0$ whenever $g \neq 0$. How does this relate to the Wronskian of f and g?

Show that the three functions $y_1(t), y_2(t) = te^t, y_3(t) = t^2e^t$ are linearly independent on $\mathbb{R}$.

Bonus: Show that any two solutions to the differential equation $ay' + by = 0$ must be linearly dependent.