# 4.3-4.5: Linear Second-Order Equations

Thursday, October 27

## 4.3: Complex Roots

Warmup: what is $e^{i\pi/3}$ in terms of sines and cosines?

Solve the initial value problem $w'' - 4w' + 2w = 0$ where $w(0) = 0, w'(0) = 1$.

## 4.4: Nonhomogeneous Equations

Explain what the solution will look like for a differential equation of the form...

- $ay'' + by' + cy = p_k(t)$, where $p_k$ is a degree k polynomial.
- $ay'' + by' + cy = p_k(t)e^{rt}$
- $ay'' + by' + cy = e^{\alpha t}\cos\beta t$

Find particular solutions to the given differential equations:

1. $y'' + 2y' - y = 10$
2. $y'' + 4y = 16t\sin 2t$

## 4.5 In One Sentence

The set of solutions to $A\mathbf{x} = \mathbf{b}$ is $\mathbf{x}_* + \mathrm{Nul}(A)$, where $\mathbf{x}_*$ is any particular solution.

Find all solutions of the equation $y'' + y = 1$.

## The Derivative Operator

Say we want to solve the differential equation $y'' + 2y' + 4y = 5\sin 3t$, and suspect that the solution will be of the form $A\sin 3t + B\cos 3t$. If $\mathcal{B} = \{\sin 3t, \cos 3t\}$ and $D$ is the derivative operator, find $[D]_{\mathcal{B}}$ and $[D^2]_{\mathcal{B}}$. Use this information to set up the problem as a system of equations $A\mathbf{x} = \mathbf{b}$.

Say we want to solve the differential equation $y'' - 2y' + y = 3e^t$. If $\mathcal{B}$ is the basis $\{e^t\}$, what does the resulting linear system look like? If $\mathcal{C}$ is the basis $\{te^t, e^t\}$, what are $[D]_{\mathcal{C}}$ and the resulting linear system?

## Linear Independence

Prove that if $T$ is a linear transformation and $\{T(v_1), \ldots, T(v_n)\}$ is a linearly independent set, then $\{v_1, \ldots, v_n\}$ is also a linearly independent set.

For $p \in \mathbb{P}_2$, define $T : \mathbb{P}_2 \to \mathbb{R}^3$ by $T(p) = [p(0), p(1), p(3)]$. Use the previous problem to show that $\{1, t, t^2\}$ is a linearly independent set.

Show that $\{\sin t, \sin 2t, \cos t\}$ is a linearly independent set.