1.1-1.2: Row Reduction and Echelon Forms

Thursday, August 25

Sets

Describe each of the following sets:

- $\mathbb{N}$
- $\mathbb{Z}$
- $\mathbb{R}$
- $\mathbb{C}$
- $\mathbb{Z} \cap \mathbb{R}$
- $\mathbb{Q} \cup \mathbb{N}$
- $\mathbb{R}^2$
- $\{n + \frac{1}{2} : n \in \mathbb{N}\}$
- $\{x : x \in \mathbb{R}, x^2 < 1\}$

Use set builder notation to describe the set of all odd integers.

Sketch the following subset of $\mathbb{R}^2$: $[(\frac{t}{2}, t + 1) : t \in \mathbb{R}]$

Row operations

What are the 3 types of elementary row operations on a matrix?

Give an example of a matrix that is in echelon form but not reduced echelon form. Give an example of a matrix whose entries are all 1 or 0 but is not in echelon form.

(1.2, Example 3): Use elementary row operations to transform the following matrix into echelon form, then reduced echelon form.

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

(1.2, Example 5): Determine the existence and uniqueness of the solutions to the system

$$\begin{aligned} 3x_2 - 6x_3 + 6x_4 + 4x_5 &= -5 \\ 3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 &= 9 \\ 3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 &= 15 \end{aligned}$$

Suppose you want to find a parabola of the form $y = ax^2 + bx + c$ that passes through the points (1,1), (-1,7), and (2,4). Set up this problem as a system of linear equations, form the augmented matrix system and transform it to reduced echelon form, and describe the set of solutions.

Do the same with fitting a parabola of the form $y = ax^2 + bx + c$ through the following sets of points:

- (-1,2), (2,3), and (2,5)
- (-2,8), (-1,4), (0,3), and (2,4)
- (-1,2) and (1,2)