Midterm 1: Review

Tuesday, September 20

1 Definitions

- (reduced) echelon form
- pivot, pivot column
- free variable, (in)consistent
- vector, scalar, linear combination, span
- parametric vector form, homogeneous equation
- linear (in)dependence
- linear transformation
- domain, codomain, range, image
- one-to-one, onto
- elementary matrix
- singular, determinant, invertible
- determinant
- vector space, subspace
- column space, null space, kernel
- basis

2 Computational Problems

- Transform a given matrix A to (reduced) echelon form.
- Find all solutions to $A\mathbf{x} = \mathbf{b}$ in parametric vector form.
- Multiply two matrices correctly, or determine whether they cannot be multiplied.
- Find the transpose of a given matrix.
- Find all solutions to $A\mathbf{x} = \mathbf{0}$ in parametric vector form.
- Determine whether $\mathbf{u} \in \mathrm{Span}(\mathbf{v}_1, \ldots \mathbf{v}_n)$.
- Determine whether a set of vectors is linearly independent.
- Determine whether a set of vectors spans a space.
- Determine whether a given function is a linear transformation.
- Determine whether a linear transformation T is 1-1 and/or onto.
- Determine whether A is invertible

- Find det(A) using cofactor expansion and/or elementary row/column operations.
- Given a shape B, find the area/volume of $T(B)$.
- Find A^{-1} using row reduction.
- Solve $A\mathbf{x} = \mathbf{b}$ using Cramer's Rule.
- Find A^{-1} using (a theorem related to) Cramer's Rule.
- Determine whether a given set is a vector space/subspace.
- Find the null space and column space of a matrix in parametric vector form.
- Determine whether a given vector is in the column space/null space of a given matrix.
- Determine whether a set of vectors B is a basis for a supspace H of a vector space V.
- Given a set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$, find a subset of the vectors that form a basis for that set.
- Given a matrix, find bases for its null space and column space.

3 Logic

- Given a statement, find its contrapositive and its negation.
- Define matrix vocabulary (e.g. 1-1, onto, consistent, span, independent, etc.) rigorously in terms of EXISTENCE and FOR-ALL statements.

4 Conceptual Ties

- Given a matrix A geometrically describe the effect of the linear transformation associated with A.
- INVERSE MATRIX THEOREM. In particular, most of what we have covered so far can be viewed from four perspectives:

  1. Pivots: what the (reduced) echelon form of A looks like
  2. Matrix equations: what is the solution set of $A\mathbf{x} = \mathbf{b}$? What is det(A)? Does A have an inverse?
  3. Vectors: Is $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ linearly independent? Does the set span a certain space? Is a particular vector $\mathbf{u}$ contained in the span? Does it form a basis?
  4. Linear transformations: What is the range of T? Is it 1-1? Onto? Given a subspace S, what can we say about the image $T(S)$?

  These perspectives are very closely related to each other. This means that many of the problems under "Compuational Problems" are really just slightly different ways of stating the same problem. Find these connections and explain why the problems are related.

- Related to the above: given a problem stated from one of the four above perspectives, rephrase it (if possible) in terms of the other perspectives.