Final Review Problems

Friday, December 2

1. Consider the second-order differential equation $y'' + \alpha y' + 9y = 0$. For what values of α will the auxiliary equation have real roots? Complex? Multiple? Find the general solution in all three cases.

2. Find the general solution to the differential equation $y'' - y' - 2y = e^{-t}$.

3. Use the method of undetermined coefficients to find a general solution to the system

$$\mathbf{x}'(t) = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \mathbf{x}(t) + e^{-2t} \begin{bmatrix} t \\ 3 \end{bmatrix}.$$

4. Use variation of parameters to find a general solution of the system

$$\mathbf{x}'(t) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

5. Find e^{At} where $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$.

6. A metal rod of length L starts at 100 degrees Celsius when suddenly the temperature at both endpoints is fixed to zero. If the rate of diffusion of heat through the rod is governed by the equation $u_t = u_{xx}$, find the temperature of the rod at any given point x and time t.

7. Find a formal solution to the vibrating string problem governed by the given set of equations:

$$u_{tt} = 9u_{xx}, 0 < x < \pi, t > 0$$
$$u(0, t) = u(\pi, t) = 0, t > 0$$
$$u(x, 0) = \sin 4x + 7\sin 5x, 0 < x < \pi$$
$$u_t(x, 0) = 0.$$