Quiz 9; Tuesday, November 8
MATH 54 with Ming Gu
GSI: Eric Hallman

Student name:

You have 20 minutes to complete the quiz. Calculators are not permitted.

1. (12 points) Solve the initial value problem

$$y'' + 2y' + 17y = 17t$$

where $y(0) = 0$ and $y'(0) = 1$.

ANSWER:

The roots of the characteristic equation are $-1 \pm 4i$, so the general solution to the homogeneous equation is of the form

$$y_h = c_1 e^{-t} \sin 4t + c_2 e^{-t} \cos 4t.$$

Use the method of undetermined coefficients to find a particular solution, and guess that $y_p = At + B$. This gives the system of equations

$$\begin{aligned} 17At &= 17t \\ 17B + 2A &= 0, \end{aligned}$$

which has the solution $A = 1, B = -2/17$, so $y_p = t - 2/17$. Note that $y_p(0) = -2/17$ and $y_p'(0) = 1$. Solving for c_1 and c_2 then gives the equations

$$\begin{aligned} c_2 - 2/17 &= 0 \\ 4c_1 - c_2 &= 0, \end{aligned}$$

which gives the overall solution

$$y = \frac{2}{17} e^{-t} \cos 4t + \frac{1}{34} e^{-t} \sin 4t + t - \frac{2}{17}.$$