Quiz 7; Tuesday, October 18
MATH 54 with Ming Gu
GSI: Eric Hallman

## Student name:

You have 15 minutes to complete the quiz. Calculators are not permitted.

1. (4 points) Find the QR factorization of the matrix $\left[\begin{array}{cc}0 & 3 \\ 1 & -2\end{array}\right]$. In other words, write the matrix as a product $Q R$, where $Q$ is orthogonal and $R$ is upper triangular.
2. (4 points) Let $\mathbf{u}$ be a unit vector in $\mathbb{R}^{2}$ and let $\mathbf{x}$ be a vector such that $\mathbf{u}^{T} \mathbf{x}=-2$ and that $\mathbf{x}$ and $\mathbf{u}$ are linearly independent. Sketch and label $\mathbf{u}, \mathbf{x}, \operatorname{proj}_{\mathbf{u}}(\mathbf{x})$, and $\mathbf{x}-\operatorname{proj}_{\mathbf{u}}(\mathbf{x})$. There may be multiple correct sketches.
3. (4 points) Mark each statement as True or False. You do not have to explain your reasoning.
(a) If $L$ is a line through $\mathbf{0}$ and $\hat{\mathbf{y}}$ is the orthogonal projection of $\mathbf{y}$ onto $L$, then $\|\hat{\mathbf{y}}\|$ gives the distance from $\mathbf{y}$ to $L$.
(b) For any subspace $W$ and vector $\mathbf{x}, \mathbf{x}-\operatorname{proj}_{W} \mathbf{x}$ must be an element of $W^{\perp}$.
(c) For any matrix $U, U U^{T} \mathbf{y}$ is the projection of $\mathbf{y}$ onto the span of $U$.
(d) If $Q$ is an orthogonal matrix then $Q^{-1}$ is also an orthogonal matrix.
