Student name:

You have 15 minutes to complete the quiz. Calculators are not permitted.

Consider the system $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t), t \geq 0$, with $\mathbf{A} = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$.

1. (6 points) Find the eigenvalues and eigenvectors of the matrix $\mathbf{A}$.

   ANSWER: The characteristic polynomial of A is $\lambda^2 - 4$, so the eigenvalues are $\lambda_1 = 2, \lambda_2 = -2$. The corresponding eigenvectors are $\mathbf{v}_1 = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -1 \\ \sqrt{3} \end{bmatrix}$.

2. (3 points) Find the general solution to the system $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$.

   ANSWER: $\mathbf{x}(t) = c_1 e^{2t}\mathbf{v}_1 + c_2 e^{-2t}\mathbf{v}_2$.

3. (3 points) Sketch the trajectory of the solution having initial vector $\mathbf{x}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

   ANSWER: something like this very professional-looking sketch. The eigenvector axes are in green and the trajectory of the solution is in blue.