Quiz 1; Tuesday, August 30
MATH 54 with Ming Gu
GSI: Eric Hallman

Student name:

You have 15 minutes to complete the quiz. Calculators are not permitted.

1. (4 points) Solve the system of equations

$$x_1 + 5x_2 = 7$$
$$-2x_1 - 7x_2 = -5$$

by setting up the augmented matrix and using elementary row operations to convert it to *reduced* echelon form. Describe which row operation you are using with every step.

$$\begin{bmatrix} 1 & 5 & 7 \\ -2 & -7 & -5 \end{bmatrix} \mapsto_{(2)=(2)+2\cdot(1)} \begin{bmatrix} 1 & 5 & 7 \\ 0 & 3 & 9 \end{bmatrix}$$

$$\mapsto_{(2)=(2)/3} \begin{bmatrix} 1 & 5 & 7 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\mapsto_{(1)=(1)-5\cdot(2)} \begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & 3 \end{bmatrix}$$

The unique solution is

$$x_1 = -8$$
$$x_2 = 3$$

2. (4 points) Describe the set of solutions for the system whose augmented matrix is given below:

$$\begin{bmatrix} 1 & 0 & -5 & 0 & 0 & 3 \\ 0 & 1 & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The set of solutions is

$$\begin{cases} x_1 & = 3 + 5x_3 \\ x_2 & = 5 - 4x_3 + x_4 \\ x_3 & \text{is free} \\ x_4 & \text{is free} \\ x_5 & = 0 \end{cases}$$

3. (4 points) Mark each statement as True or False. You do not have to explain your reasoning.

(a) Every elementary row operation is reversible. TRUE

(b) Elementary row operations on an augmented matrix never change the solution set of the associated linear system. TRUE

(c) If one row of an echelon form of an augmented matrix is $\begin{bmatrix} 0 & 0 & 0 & 5 & 0 \end{bmatrix}$, then the associated linear system is inconsistent. FALSE: it just implies that $x_4 = 0$.

(d) Whenever a system has free variables, the solution set contains infinitely many solutions. FALSE: it might be inconsistent. For example, the system

$$x + y = 5$$
$$0 = 1$$

has y as a free variable but has no solutions.