Chapter 7.1-2: Probability!

Wednesday, October 21

Summary

- Probability of an event $E \subset S$: $p(E) = \sum_{s \in E} p(s)$.
- Uniform distribution: if $|S| = n$ then $p(s) = 1/n$ for all $s \in S$.
- $p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$
- Conditional probability: $p(E|F) = \frac{p(E \cap F)}{p(F)}$.
- E, F independent if $p(E \cap F) = p(E) \cdot p(F)$.
- $E_1, \ldots, E_n$ mutually independent if $p(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{im}) = p(E_{i_1})p(E_{i_2}) \cdots p(E_{i_m})$ for any subset $\{i_1, \ldots i_m\} \subset \{1, \ldots, n\}$.

Warmup

Suppose we roll a pair of fair six-sided dice.

1. What is the chance of rolling a seven?
2. What is the chance of *not* rolling a seven?
3. What is the chance of rolling an odd number *or* a number divisible by 3?
4. Say we roll the dice one at a time. If the first die lands on a 4, what is the chance that the second die will also land on a 4?

Uniform Probability

1. What is the probability that a 5-card hand drawn from a well-shuffled deck contains five cards of the same suit?
2. Four friends are playing a game of Monopoly and want to roll the dice to decide who goes first. Find a way to decide who goes first that (1) requires only one roll of a pair of dice, and (2) gives every player an equal probability of going first.
3. Which is more likely: rolling an 8 when a total of 2 dice are rolled or rolling a total of 8 when three dice are rolled?
4. Which is more likely: flipping exactly 5 heads out of 10 or flipping exactly 10 heads out of 20?

Conditional Probability and Independence

1. You roll 2 dice. What is the probability that the sum is 7 if *at least* one of the two numbers is a 5?
2. You roll 2 dice. What is the probability that the sum is 7 if *exactly* one of the two numbers is a 5?
3. You flip 4 coins. What is the probability that you have flipped four heads if at least three of the coins landed on heads?
4. You flip 4 coins. What is the probability that you have flipped four heads if *the first* three coins landed on heads?
5. If E and F are independent events, prove or disprove that $\overline{E}$ and F are independent events.
6. A three-member jury has two members who make the correct decision with probability p and one member who flips a fair coin. A one-member jury will make the correct decision with probability p. Which jury is more likely to make the correct decision?
7. Two coins are flipped. Let A be the event that the first coin lands on heads, let B be the event that the second coin lands on heads, and let C be the event that an even number of heads is flipped. Show that A, B, and C are pairwise independent but not mutually independent.