Chapter 6.5: Multinomials, Stars-and-Bars

Monday, October 19

Error Spotting

Of the solutions given to the following problems, determine which ones are correct and explain the errors in the incorrect ones.

1. How many strings of 5 digits (0-9) are there with at least one 7?

   (a) The number of strings with at least one 7 is equal to the total number of strings minus those with no 7s, which is $10^5 - 9^5$.

   (b) Fix one of the 5 digits and make it a 7. There are then 10 choices for each of the remaining digits, so the correct answer is $5 \cdot 10^4$.

   (c) To get the number we add the number of strings with exactly 1, 2, 3, 4, and 5 sevens: thus the answer is $1 \cdot 9^4 + 1 \cdot 1 \cdot 9^3 + 1^3 \cdot 9^2 + 1^4 \cdot 9 + 1^5$.

2. How many strings of 5 digits have exactly two odd digits and exactly one digit greater than 5?

   (a) There are $\binom{5}{2}$ places to put the odd digits and 5 choices for each odd digit. Then there are 3 places to put the large digit and 4 choices (6,7,8,9) for that digit. The other two digits must be 0,2, or 4, so this gives $\binom{5}{2} \cdot 5^2 \cdot 3 \cdot 5 \cdot 3^2$ choices in all.

   (b) If the digit greater than 5 is even then we have 5 choices each for the odd digits (1,3,5,7,9), 2 choices for the even digit (6,8), and three choices for the other two digits (0,2,4). If the large digit is odd then we have 3 choices for the small odd digit (1,3,5), two for the large odd digit (7,9), and three each for the other three digits (0,2,4). The answer is the sum of these two numbers times the 5! ways to permute the five digits, so $(5^2 \cdot 2 \cdot 3^2 + 3 \cdot 2 \cdot 3^3) \cdot 5!$

Miscellany

1. How many ways to put 7 identical balls in 4 identical bins?
2. How many ways to put 7 identical balls in 7 identical bins?
3. How many ways to put 7 identical balls into a red bin, a blue bin, and a green bin?
4. You have a 52-card deck. How many ways can you deal 5 cards to each of 6 players?
5. How many ways can you deal 13 cards to each of 4 players?
6. How many ways can you deal 13 cards to each of 4 players so that each player gets one card of each of the 13 values (ace-2-3-…-king)?
7. How many ways can you give 10 cookies to 4 friends if each friend gets at least 1 cookie?
8. How many 5-digit sequences have the digits in non-decreasing order?
9. For how many 5-digit sequences are the digits in *strictly* increasing order (so no repetitions)?
10. How many ways can you split nine (non-identical) people into three (identical) groups of three?

Poker

Say you draw a 5-card hand for a round of poker. You have a *full house* if you have three cards of one value and two of another (e.g. 5-5-5-7-7). You have a *straight* if the five cards are all consecutive values (e.g. A-2-3-4-5 or 10-J-Q-K-A, aces can count both low and high). You have a *flush* if all five cards are of the same suit (e.g. $3\heartsuit - 5\heartsuit - K\heartsuit - 10\heartsuit - 7\heartsuit$). A hand that is both a straight and a flush is a *straight flush*, and it is much more valuable than either a straight or a flush.
Your task: find the number of ways to draw a full house, straight, and flush (excluding straight flushes). Rank the three hands from most valuable to least valuable.