

# Chapter 6.1-6.2: Counting

Monday, October 12

## Warmup

1. Define a sequence  $a_n$  by  $a_0 = 0$ ,  $a_1 = 1$ , and for  $n \geq 2$ ,  $a_n = 2 \cdot a_{n-1} - a_{n-2}$ . Find a non-recursive formula for  $a_n$  and prove that it is correct.
2. How many numbers between 1 and 60 are divisible by 2?
3. How many are divisible by 3?
4. How many are divisible by 5?
5. How many are divisible by 2 or 3 or 5?

## Inclusion-Exclusion

1. How many numbers between 1 and 1000
  - (a) Are divisible by both 7 and 11?
  - (b) Are divisible by either 7 or 11?
  - (c) Are divisible by 7 but not by 11?
  - (d) Are divisible by neither 7 nor 11?
2. 36 students go to a hot dog stand and order hot dogs. Every student orders at least one topping. You have the following information about their topping choices:

(a) 18 ask for mustard.	(e) 31 ask for onions or relish (or both).
(b) 21 ask for onions.	(f) 17 ask for exactly two toppings.
(c) 18 ask for relish.	(g) 2 ask for all three toppings.
(d) 8 ask for mustard but not onions.	

How many students order exactly one topping? (Try making a Venn diagram.)

## Pigeonhole Principle

1. It is said of the town of Lake Wobegon that “all the women are strong, all the men are good looking, and all the children are above average.” Discuss.
2. Come up with a related formal statement and prove it. How does this relate to the Pigeonhole Principle?

## Tree Diagrams

1. Use a tree diagram to find the number of subsets of  $\{3, 7, 9, 11, 24\}$  such that the sum of the elements in the subset is less than 28.

## Division Rule and Symmetries

1. How many ways are there to seat 5 people around a circular table?
2. If we make a 4-sided die out of a tetrahedron (4 faces, all equilateral triangles), then how many possible arrangements of the numbers are there?
3. How many possible arrangements are there on a 6-sided die?