Chapter 5.1: Induction and Recursion

Wednesday, September 30

Warmup

Define: A *curious* number is a number that is curious. Suppose we know two things about curious numbers:

1. If any integer n is a curious number, then $n+2$ is a curious number.
2. 7 is a curious number.

Which numbers *must* also be curious?

1. 5
2. 9
3. 10
4. 15
5. 1341
6. 2808
7. 39523092357
8. n
9. ∞

Now suppose that 10 is *not* a curious number. Which number must *not* be curious?

There is a machine that makes widgets all day. It has one problem— if a widget it makes is defective, then the next widget it makes will also be defective. What can you say about the machine?

Induction

1. For what integers is $2^n \geq n^2$ true? Prove it.
2. (Calculus) Suppose we know that $\frac{d}{dx}x = 1$ and that for any functions f and g, $(fg)' = f'g + fg'$. Prove that $\frac{d}{dx}x^n = nx^{n-1}$ for all $n \geq 1$.
3. Prove that $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$ for $n \geq 0$.
4. Prove that $1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n+1)! - 1$ for $n \geq 1$.
5. Find a closed form for $\sum_{k=1}^{n}(-1)^k k^2$ and prove that it is correct.