Chapters 4.1-4.2: Number Theory

Wednesday, September 16

Key Notes

- $a|b = a$ divides $b = a$ is a divisor of $b = b$ is divisible by $a = (\exists k \in \mathbb{Z})(ak = b)$
- $a \equiv b \pmod{m}$ if and only if $m|(b-a)$.
- If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ then $a + c \equiv b + d \pmod{m}$ and $ac \equiv bd \pmod{m}$.
- If $a|b$ and $a|c$ then $a|(mb + nc)$ for any $m, n \in \mathbb{Z}$.

Warmup

1. Today is Tuesday. What day will it be 1000 days from now?
2. You are on a circular track 400 meters long. You run 3800 meters clockwise and 2200 meters counter-clockwise. How far are you from where you started?
3. Observations: If $30|n$ then $10|n$. If $25|n$ then $5|n$. If $18|n$ then $9|n$. Find a general rule.

Modular Arithmetic

Evaluate the following:

1. $44 \pmod{3}$
2. $171 \pmod{12}$
3. $-26 \pmod{5}$
4. $199^2 \pmod{5}$
5. $(2301 \pmod{3})^2 \pmod{5}$
6. $23^{88} \pmod{2}$
7. $2^{100} \pmod{10}$
8. $2737 \cdot 8184 \pmod{9}$
9. $2^{64} \pmod{13}$
10. $88^5 \pmod{90}$
11. $97 \cdot 85 \pmod{100}$
12. $155 \cdot 822 \pmod{10}$

Divisibility

True or false? If true, prove. If false, find a counterexample.

1. $1|a$ for any a.
2. $0|a$ for any a.
3. $a|0$ for any a.
4. If $a|b$ and $b|c$ then $a|c$.
5. If $a|b$ and $b|a$ then $a = b$.
6. If $a|c$ and $b|c$ then either $a|b$ or $b|a$.
7. Suppose $a|b$. Then $a|(b + c)$ if and only if $a|c$.
8. If $2|n$ and $4|n$ then $8|n$.

Divisibility Tests

1. Prove that a number is divisible by 5 if and only if its last digit is 0 or 5.
2. Prove that a number is divisible by 4 if and only if its last two digits make a number divisible by 4.
3. Prove that for any integer n, either $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$.
4. Prove that 98765434 is not a perfect square.
5. Prove that 111111 cannot be written as the sum of any two square numbers (what are the possibilities for $a^2 + b^2 \pmod 4$?)