# Chapters 2.2-2.5: Sets and Functions

Monday, September 14

## Key Notes

- One way to prove $A = B$: prove $A \subset B$ and $B \subset A$.
- $f : A \to B$ is 1-1 if $f(a) = f(b) \Rightarrow a = b$ for all $a, b \in A$.
- $f : A \to B$ is 1-1 if $a \neq b \Rightarrow f(a) \neq f(b)$ for all $a, b \in A$.
- $f : A \to B$ is onto if $(\forall b \in B)(\exists a \in A)(f(a) = b)$.

## Warmup

1. What rule in propositional logic does $\overline{A \cup B} = \overline{A} \cap \overline{B}$ correspond to?
2. How can you say "$f$ is not onto" in quantifier notation? How do you prove that a function is not onto?

## Set Proofs 1

Consider the statement "If $A \subseteq B$ then $A \cup C \subseteq B \cup C$."

1. Find an analogous statement in propositional logic.
2. Draw a Venn Diagram that illustrates the proof.
3. Make a given-goal diagram for the proof.
4. Prove the statement. Prove or disprove the converse.

Do the same with the statement "If $A \subset B$ then $B \cup \overline{A} = U$."

## Set Proofs 2

Prove the following statements using a series of equivalences.

1. $A - (A - C) \subseteq C$
2. $\overline{A \cup B} \cap B = \emptyset$
3. $B = (A \cap B) \cup (\overline{A} \cap B)$

## Functions

1. Prove that $f(x) = 3x$ is onto if f is from $\mathbb{R}$ to $\mathbb{R}$ but not if $f$ is from $\mathbb{Z}$ to $\mathbb{Z}$.
2. Prove that $f(x) = 3x$ is 1-1 in either of the above cases.
3. Say that a function is *increasing* if $a > b$ implies that $f(a) > f(b)$ for all $a, b$ in the domain of $f$. Prove that increasing functions are 1-1.
4. Prove that the sum of two increasing functions is increasing.
5. Prove or disprove: the sum of two 1-1 functions is 1-1.

## Cardinality

1. Prove that if $A$ and $B$ are countable then $A \cup B$ is countable.
2. Prove that if $A$ is countable and $B$ is uncountable then $B - A$ is uncountable.