Chapter 10.7: Planar Graphs

Wednesday, December 2

Summary

- A *planar graph* can be drawn in the plane so that no edges intersect. Such a drawing is a *plane graph*.
- Euler's Formula: For a plane graph, $v - e + r = 2$.
- If $v \geq 3$ then $e \leq 3v - 6$.
- Every planar graph has a vertex of degree ≤ 5.
- If G is triangle-free and $v \geq 3$ then $e \leq 2v - 4$
- Kuratowski's Theorem: a graph is planar if and only if it contains no subgraph homeomorphic to K_5 or $K_{3,3}$.

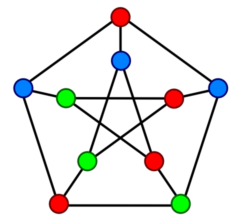
Euler's Formula

1. A connected plane graph has six vertices, each of degree four. Into how many regions does the graph divide the plane?
2. Find all five Platonic solids. Each one can be described by gluing regular $n-$gons together with m polygons to a vertex. For example, a cube is made of squares with 3 squares to a vertex and has 6 faces. Find the number of faces of the other Platonic solids.
3. If you have a solid block made of triangles and octagons with 3 shapes to a vertex, what can you say about the relation between the number of triangles and the number of octagons? (One of these shapes is made by slicing the vertices off of a cube.)
4. What if you have a solid block made of triangles and squares with 4 shapes to a vertex?
5. Find two trees with the same number of vertices that are homeomorphic but not isomorphic.

Planar Graphs

1. Show that if a connected graph has n vertices and $n + 2$ edges, then G is planar. For $n \geq 6$, this becomes false if we say $n + 3$ instead of $n + 2$.

2. Show that the Petersen graph is not planar (find a subgraph homeomorphic to $K_{3,3}$).



3. Find a bipartite graph G that is not planar but does NOT contain a subdivision of $K_{3,3}$. Your graph should have no more than 20 edges.

Bonus

1. Solve the problem that Matt Damon solved in *Good Will Hunting*: "How many (non-isomorphic) homeomorphically irreducible trees are there on 10 vertices?" A tree is *homeomorphically irreducible* if it has no vertices of degree 2.

2. If $v - e + r = k$ for a tiling of some shape, then we can say that the *Euler characteristic* of the shape is k. For example, the Euler characteristic of a sphere is 2. Do some experimentation and find the Euler characteristics of the torus and double torus.