

# Chapter 10.3-10.4: Paths and Isomorphisms

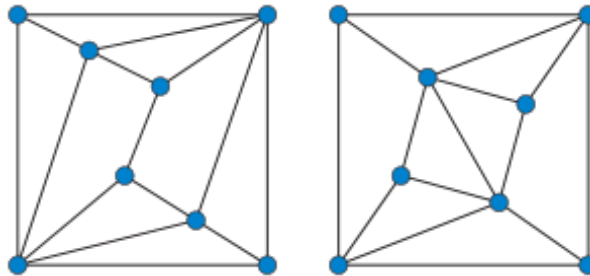
Monday, November 30

## Definitions

- $\kappa(G)$ : Vertex connectivity of  $G$ .
- $\lambda(G)$ : Edge connectivity of  $G$ .

## Isomorphisms

1. Show that  $C_5$  and  $\overline{C_5}$  are isomorphic.
2. Show that  $C_4$  and  $\overline{C_4}$  are not isomorphic.
3. Find a graph  $G$  on 4 vertices such that  $G$  and  $\overline{G}$  are isomorphic.
4. Find all isomers (non-isomorphic graphs) of pentane ( $C_5H_{12}$ ).
5. Show that the following two graphs are not isomorphic:

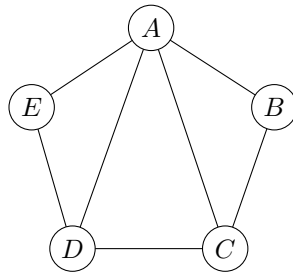


## Connectedness

1. Find a graph  $G$  such that  $\kappa(G) < \lambda(G)$ .
2. Find a graph  $G = (V, E)$  such that  $\lambda(G) < \min_{v \in V} \deg(v)$ .

## Adjacency Matrices

Here is a picture of a graph:



1. Draw the adjacency matrix  $A$  of the graph.
2. Find  $A^2$ . What do the diagonal elements of the matrix tell you?
3. How can you use  $A^3$  to count the triangles in a graph?

## Proofs

1. Show that if  $G$  and  $\overline{G}$  are isomorphic then  $n \equiv 0 \pmod{4}$  or  $n \equiv 1 \pmod{4}$ .
2. Prove: if  $v$  has odd degree in  $G$  then there is some vertex  $w$  such that  $v$  and  $w$  are connected by a path.