Chapter 8.4-6: Generating Functions and Inclusion-Exclusion

Monday, November 16

Warmup

1. $\frac{1}{1-x} = \sum_{i=0}^{\infty}$
2. $\frac{1}{1-2x} = \sum_{i=0}^{\infty}$
3. $\frac{1}{1-x^2} = \sum_{i=0}^{\infty}$
4. $\sum_{i=0}^{\infty}(n+1)x^n =$
5. If E and F are independent events, what is $p(E \cup F)$?

Generating Functions

1. How many ways are there to make change for a dollar with pennies, nickels, dimes, quarters, and half-dollars? Do not find the answer, but explain how to get it using generating functions.
2. Give eight cookies to three children so that each child gets between 1 and 4 cookies.
3. Find a function that generates the sequence $a_n = n \cdot 3^n$.

More Inclusion-Exclusion

1. How many numbers between 1 and 60 are divisible by 2 or 3 or 5?
2. Four men check four hats, which at the end of the evening are returned to them randomly. For each number n between 0 and 4, find the probability that n of the men get their correct hat back.

Relations

Decide whether each of these relations is reflexive, symmetric, antisymmetric, or transitive:

1. $(a, b) \in R$ if $a \geq b$.
2. $(a, b) \in R$ if $a = b$.
3. $(a, b) \in R$ if $ab = 0$.
4. $((a/b), (c, d)) \in R$ if $ac = bd$.

Draw a Venn Diagram with a circle for each of the three properties "reflexive," "symmetric," and "transitive." There are eight regions in this Venn Diagram. Find a relation that belongs in each region.